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MULTIQUARK COLOUR-HYPERFINE SPECTRA IN THE BAG MODEL: A SURVEY WITH THE P-MATRIX FORMALISM

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CONTENTS

The masses of S-wave $q^2\bar{q}^2$, $q^4\bar{q}$ and q^6 multiquark states have been calculated in the M.I.T. bag model after taking proper account of flavour symmetry violations in the single-gluon-exchange magnetic contribution. Mixing induced by the flavour dependence of the colour-magnetic interaction strengths raises some degeneracies and inverts a few levels but does not otherwise greatly affect the masses. However, the mixing does in a few cases result in very substantial changes in the eigenfunctions. These effects are surveyed and the masses and dissociation couplings for the most important states are examined using the P-matrix formalism.

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1. Introduction

Six years ago Jaffe (1977*a*, *b*) – and in an earlier preliminary report, Jaffe & Johnson (1976) – suggested that the 0^{++} mesons $\varepsilon(700)$, $S^*(980)$, $\delta(980)$ and (the now defunct) $\kappa(800\text{–}1100)$ were $q^2\bar{q}^2$ states and thus revived considerable interest in multiquark hadrons. Jaffe's contention was based on calculations carried out using the spherical cavity approximation to the M.I.T. bag model. All parameters in this model had been fixed by fits to the meson and baryon spectrum (DeGrand *et al.* 1975) and the result was that the masses of $q^2\bar{q}^2$ states turned out to be quite low – some less than 1 GeV – and within experimentally accessible regions. It was also found that those with quantum numbers shared by ordinary hadrons, which Jaffe termed 'cryptoexotics' (C), were lower in mass than those with exotic (E) quantum numbers. This was a consequence of the colour-hyperfine structure arising from single-gluon-exchange magnetic interactions and thus the result was more general than the bag model. Similar results were deduced (Jaffe 1977*c*, *d*) for the $q^4\bar{q}$ and q^6 configurations.

Since Jaffe's early work there has been an increasing realization (Jaffe 1978, 1979) that the interpretation initially assumed, of bag model multiquark eigenstates, is theoretically unsound. The spherical cavity approximation to the bag model artificially confines all quarks inside the bag but, unlike mesons and baryons, multiquark hadrons can conceivably reconstitute themselves into colour neutral components and absolute colour confinement can then no longer be invoked to ensure binding. Originally it was assumed (Jaffe 1977*a*) that this possibility of dissociation would be manifested as an 'OZI-superallowed' decay resulting in broad widths for multiquark resonances (cf. Okubo 1963; Zweig 1964; Iizuka 1966). However, the concept of a decay involves the existence, in the first instance, of a quasi-stationary state. If the bag eigenstates are simply artefacts of the artificial boundary condition then they do not correspond to particle resonances.

An alternative interpretation is provided by the P-matrix formalism of Jaffe & Low (1979). These authors show how to construct from experimental phase shifts a quantity, termed by them the P matrix, which is argued to have poles at precisely those energies where the spherical cavity bag Hamiltonian possesses eigenstates. They were able to determine several experimental pole positions in meson–meson channels and found remarkable agreement with the bag model $q^2\bar{q}^2$ predictions. Their interpretation of the spherical cavity eigenstates becomes particularly appealing when one considers the uncertain resonant nature of the scalar mesons. Indeed the $\kappa(800\text{–}1100)$, acknowledged as doubtful in Jaffe's (1977*a*) paper, is now considered to be quite 'dead' and the broad $\varepsilon(700)$, also acknowledged there as doubtful, remains under considerable suspicion, it being no longer included in the Particle Data Group (1982) tabulation, although it cannot be ruled out. Even the rather narrow and well established S^* and δ seem to be largely a consequence of the proximity of the $K\bar{K}$ threshold. (For a recent analysis see Törnqvist (1982).) The absence of the $\kappa(800\text{–}1100)$ is a serious mark against the early naïve interpretation of the bag model eigenstates as there is no obvious way to remove an isospin $I = \frac{1}{2}$, strangeness $S = +1$ state in this mass range from the bag model spectrum. On the other hand, one finds a clear P-matrix pole in reasonable agreement with the predictions regardless of the non-resonant nature of the πK phase shift. P-matrix poles are even found in exotic channels, in agreement with bag model predictions, although the phase shifts are definitely non-resonant. An investigation by Roiesnel (1979) of P-matrix poles in the $q^4\bar{q}$ sector, based on the work of Strottman (1979), again shows reasonable agreement with what is expected from the bag model. Jaffe & Shatz (1980)

have looked at the P matrix in the q^6 nucleon–nucleon channels where final state interactions were thought to be important (Jaffe & Low 1979). With particular reference to this case, but on quite general grounds, they convincingly argue that the spherical cavity eigenstates can have no direct relation with loosely bound states such as the deuteron. This clarifies another bag model puzzle arising from the 'prediction' (Jaffe 1977*d*) of a 'deuteron' 280 MeV too high in mass with an 80% exotic colour content (Matveev & Sorba 1978). This bag model eigenstate is now recognized as having no direct relation with the deuteron and in fact agrees fairly well with an observed P-matrix pole.

Besides the interpretation, an unsatisfactory feature of multiquark bag model calculations has been the treatment of the single-gluon-exchange contributions. The dominant term here is a colour magnetic interaction (DeGrand *et al.* 1975) leading to 'hyperfine' splittings, analogous to the spin–spin interaction in atomic physics. Technical difficulties in evaluating this term have forced the use of an approximation (Jaffe 1977*a*, *b*) whereby a radial integral incorporating the magnetic interaction strengths is averaged over flavour quantum numbers. While this gives a fair approximation to the eigenenergies it has the undesirable effect of creating degeneracies (Jaffe 1977*a*, *b*).

One should note here the work of the Nijmegen group (Aerts 1979; Aerts *et al.* 1978, 1980; Mulders 1980; Mulders *et al.* 1978, 1979, 1980) who have actually gone beyond the S-wave ground states and considered orbital and radial excitations as well. While they have attempted (Aerts *et al.* 1978; Aerts 1979) to calculate the colour-hyperfine term without using the 'Jaffe approximation', the magnitude of calculations with their techniques prevented them from obtaining a complete solution. Therefore, they mostly use a similar averaging procedure (Mulders *et al.* 1979) to Jaffe. In addition, they use an average radius (Aerts *et al.* 1978; Mulders *et al.* 1978) for the various configurations, rather than separately minimizing the energy of each state with respect to radius (DeGrand *et al.* 1975). (Actually, this approximation is fairly good and has the advantage of greatly simplifying their mass calculations.)

The author and B. G. Wybourne have previously shown (Bickerstaff & Wybourne 1980) how to use modern group theoretic techniques to evaluate precisely the colour-magnetic term. These techniques involve writing the colour-spin operator

$$\Delta^{ab} = -\sum_{i>j}^{a,b} (\boldsymbol{\sigma}\boldsymbol{\lambda})_i \cdot (\boldsymbol{\sigma}\boldsymbol{\lambda})_j \qquad (1.1)$$

(where i and j are quarks of flavour a and b respectively and $\boldsymbol{\sigma}$ and $\boldsymbol{\lambda}$ are generators of spin and colour) as a scalar coupled product of $SU_2^J \times SU_3^C$ tensor operators, just as the spin–spin interaction in atomic physics can be conveniently expressed as a scalar coupled product of the usual angular momentum tensor operators, $\boldsymbol{T}^k$ with $k = 1$ (see, for example, Judd 1963). We have also shown (Bickerstaff & Wybourne 1981) how one may consistently calculate the projection of the resulting bag model eigenstates onto a dissociated basis. The need for phase consistency between these two calculations cannot be over emphasized and the unfortunate lack of it has been the bane of much of the early work (Wong & Liu 1980; Bickerstaff 1982). Part and parcel of our approach is the calculation of symmetrized coupling and recoupling coefficients (3*jm* factors and 6*j* symbols) for the relevant groups and subgroup schemes. The methods by which these were obtained can be found in (Bickerstaff & Wybourne 1981) and references therein. A reasonably extensive computer tabulation, supplementary to the (hand produced) tables in Bickerstaff & Wybourne (1981) is now available (Bickerstaff *et al.* 1982*a*).

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Using these methods we have been able to calculate the masses of all S-wave $q^2\bar{q}^2$, $q^4\bar{q}$ and q^6 multiquark hadrons and to provide sufficient information to dissociate all but the strangeness, $S = -1, 0$ states in the q^6 sector. Black & Wybourne (1981) have even been able to compute the entire $q^3\bar{q}^3$ spectrum this way. The magnitude of that problem can be appreciated by observing that they had to calculate matrices as large as fourteen dimensional.

A summary of the results for the $q^2\bar{q}^2$, $q^4\bar{q}$ and q^6 configurations is presented in this paper, showing for the first time the actual effects of removing the Jaffe approximation. We begin by looking at some salient features of the P-matrix formalism and by briefly outlining the relevant details of our mass calculations. Then we move on to discuss the $q^2\bar{q}^2$, $q^4\bar{q}$ and q^6 spectra. The most important low-lying states are considered in the context of the P matrix.

2. P-matrix formalism

The current definition (Jaffe & Low 1979) of the P matrix is in terms of a non-relativistic parametrization of the exterior radial scattering wave function of an n-channel two-hadron system. Here, exterior means outside a relative separation $r = b$ in which region the two hadrons are non-interacting. The true nature of the interaction potential inside b is unknown. One notes (Jaffe & Low 1979; Low 1982) that a pole in the P matrix corresponds to a state for which the exterior wave function vanishes at $r = b$. By continuity the interior wave function also vanishes, corresponding therefore to confinement by an infinite square-well potential. Jaffe and Low term such a state a primitive.

The S-wave P matrix is related to the S matrix by (Jaffe & Low 1979; Low 1982)

$$S = -\mathrm{e}^{-\mathrm{i}kb}\,\frac{1-(\mathrm{i}/\sqrt{k})\,P(1/\sqrt{k})}{1+(\mathrm{i}/\sqrt{k})\,P(1/\sqrt{k})}\,\mathrm{e}^{-\mathrm{i}kb}, \tag{2.1}$$

where k is the diagonal matrix of channel momenta. In the one-channel case, since $S = \mathrm{e}^{2\mathrm{i}\delta(k)}$ where $\delta(k)$ is the phase shift, solving (2.1) yields

$$P = k\cot[kb+\delta(k)]. \tag{2.2}$$

Explicit expressions for the two-channel P matrix are given by Jaffe & Low (1979). It is of some importance to note that when $\delta = 0$ (i.e. the no interaction case), the one-channel P matrix has a pole at

$$k_{\mathrm{c}} = \pi/b. \tag{2.3}$$

Jaffe and Low call the energy corresponding to k_{c} the 'compensation' energy, E_{c}. If $\delta > 0$ (attractive hadron–hadron potential) then the first pole in P is at $k < k_{\mathrm{c}}$ whereas if $\delta < 0$ (repulsive potential) then the first pole is at $k > k_{\mathrm{c}}$. Obviously, E_{c} depends on the value of b used and in our case this will come from the bag model.

The residues of the P-matrix poles play an important role in the phenomenology. If there is a pole at $s = s_0(b)$, where s is the usual channel invariant, then one finds (Jaffe & Low 1979; Low 1982) that the pole residue is given by

$$r(b)_{\alpha\beta} = \lambda(b)\,Q_{\alpha\beta}, \tag{2.4}$$

where

$$\lambda(b) = -\partial s_0/\partial b, \tag{2.5}$$

and (barring accidental degeneracy) $Q = Q^2$ is a projection operator that can be expressed by

$$Q_{\alpha\beta} = \xi_\alpha \xi_\beta. \tag{2.6}$$

Here, ξ_α is the (physical) coupling of the primitive to the open scattering channel, α. Of course, in the single-channel case $Q = 1$. It is expected that if the primitive is largely a creation of the artificial confinement at $r = b$ then s_0 will be very sensitive to changes in b whereas if there really exists a physical barrier, at $R_B < b$, that produces confinement largely of its own accord, then moving the artificial barrier at b further outwards should cause little change in s_0. Thus the residue should be a measure of the presence or absence of a physical barrier. It must be emphasized however (Jaffe & Shatz 1980) that the condition $b > R_B$ can only be reasonably satisfied for a deeply bound system.

It is of interest to enquire what happens if we use a single-channel P matrix in a two-channel problem below the second threshold. Jaffe and Low show that the effect of a nearby closed channel is to produce an effective open channel P matrix, $\tilde{P}$, with displaced poles and residues. Low (1982) also notes that more than one nearby primitive per channel implies some anomalous amplitude behaviour, that is, they cannot all be artefacts of a boundary condition.

It should be understood that this formalism is only defined for two-body channels and thus is only useful at low energies before the proliferation of multibody thresholds. Even so, the regions of interest require use of relativistic dynamics and this is expected to induce some small errors (Roiesnel 1979) in what is currently only a non-relativistic formalism. Furthermore, as we shall be using values of b corresponding to hadronic sizes where we know that final state interactions exist, it is essential for the sucessful application of the formalism that predictions made without taking these into account will not be unduly modified by their inclusion. Fortunately it appears (Jaffe & Low 1979; Jaffe & Shatz 1980) that this is usually the case.

(*a*) *Connection with bag model*

To apply the formalism we must now consider the connection with the bag model. This connection is by no means obvious as, although the bag model is a covariant version of an infinite square-well potential, the bag boundary conditions do not involve the vanishing of the quark wave functions. Indeed they are non-zero on the surface. Jaffe & Low (1979) noted however that the bag confinement of hadronic matter should still imply the vanishing of a hadron–hadron wave function at some relative separation $b(R_0)$ where R_0 is the equilibrium bag radius. By calculating the effective two-body density in the spherical bag and the density for a two-hadron wave function with its first zero at $r = \mathrm{b}$ and then equating the root-mean-square values of the relative separation they showed that

$$b = cR_0, \tag{2.7}$$

where c depends on the number of quarks and the hadrons involved. Specifically, one finds that

$$\langle r^2\rangle_{\substack{\text{two-hadron}\\ \text{wave function}}} = 0.283\,b^2, \tag{2.8}$$

while

$$\langle r^2\rangle_{\text{bag clusters}} = n_1^{-2}\sum_{i=1}^{n_1}\langle r_i^2\rangle + n_2^{-2}\sum_{j=1}^{n_2}\langle r_j^2\rangle, \tag{2.9}$$

where n_1 and n_2 are the numbers of quarks (plus antiquarks) in the two dissociating hadrons and $\langle r_i^2\rangle$ is the mean-square radius of a single quark in the lowest S-wave bag eigenstate and is given in table 1 of DeGrand *et al.* (1975). For massless quarks we have

$$\langle r_i^2\rangle = 0.531R_0^2, \tag{2.10}$$

and thus (Mulders 1982*b*)

$$c = 1.37(n/n_1n_2)^{\frac{1}{2}}, \tag{2.11}$$

where n is the total number of quarks. If all quarks are of flavour s then the coefficient in equation (2.11) is about 10 % smaller. When a mixture of flavours is present the decrease in c is less, but the expression for c is rather messy and may even depend on the dissociation channel, i.e. whether the s quarks appear in cluster 1 or 2. For our purposes, it will suffice to take $c = 1.4$, 1.25 and 1.1 for $q^2\bar{q}^2$, $q^4\bar{q}$ and q^6 respectively (Jaffe & Low 1979; Roiesnel 1979; Jaffe & Shatz 1980). The decrease in c with the number of quarks is easily understood by considering that the larger the number of quarks involved, the more likely it is that a hadronic subsystem will be located near the centre of the bag.

As R_0 varies from primitive to primitive it is useful to be able to relate b directly to the energy, $E_0 = s_0^{\frac{1}{2}}$, of the primitive. (Indeed if one is searching for two primitives, in the same channel but at different energies, it is almost essential to do this.) The required relation is obtained from the bag model virial theorem

$$E_0 = 4BV_0, \tag{2.12}$$

where B is the bag constant and V_0 the volume of the spherical bag occupied by the primitive. However, this theorem is only exactly true for massless quarks and the values of R_0 obtained from a specified E_0 could be in error by in excess of 20 % if s quarks are present. A better relation is readily obtained as follows. The static spherical cavity Hamiltonian can be written as

$$H = BV + P/R, \tag{2.13}$$

where

$$P = -Z_0 + \sum_a n_a \omega_a + \alpha_c \sum_{a \geqslant b} \Delta^{ab} M_{ab}, \tag{2.14}$$

and Z_0 is the 'zero-point' constant (DeGrand *et al.* 1975; cf. Milton 1980*a*, *b*; Donoghue & Johnson 1980), n_a the number of quarks of flavour a, $\omega_a \equiv \omega(m_a R)$ the $S_{\frac{1}{2}}$ eigenfrequencies of quarks of mass m_a in a spherical cavity, $\alpha_c \equiv g^2/4\pi$ the colour fine-structure constant and $M_{ab} \equiv M(m_a R, m_b R)$ the colour-magnetic interaction strengths. If only u or d quarks are present then P is a constant because $m_u = m_d = 0$ in the M.I.T. model (DeGrand *et al*, 1975). Hereafter, for convenience we will denote them collectively as ordinary, o. However, the presence of s quarks will induce an R-dependence. Because ω_a and M_{ab} are nearly linear (DeGrand *et al.* 1975) this dependence can be simply approximated by

$$P = P_{\text{massless}} + n_s (\mathrm{d}\omega_s/\mathrm{d}R)\, R + \alpha_c [\Delta^{os}\, \mathrm{d}M_{os}/\mathrm{d}R + \Delta^{ss}\, \mathrm{d}M_{ss}/\mathrm{d}R]\, R. \tag{2.15}$$

Clearly, to a good approximation the equilibrium radius is the same as in the massless case, i.e.

$$R_0 = (P_{\text{massless}}/4\pi B)^{\frac{1}{4}}, \tag{2.16}$$

and there results a straightforward correction to (2.12). We further note though that the derivatives appearing in (2.15) are approximately given by

$$\left.\begin{aligned} \mathrm{d}\omega_s/\mathrm{d}R &= 0.19\,\text{GeV}, \\ \mathrm{d}M_{os}/\mathrm{d}R &= -0.006\,\text{GeV}, \\ \mathrm{d}M_{ss}/\mathrm{d}R &= -0.010\,\text{GeV}, \end{aligned}\right\} \tag{2.17}$$

and thus the colour-hyperfine correction term can usually be neglected. The errors introduced by this approximation are normally very small and even for the most extreme values of Δ are only about 5 %. Hence we find

$$b \approx c[3(E_0 - 0.19\, n_s)/(16\pi B)]^{\frac{1}{3}}\,\text{GeV}^{-1}. \tag{2.18}$$

Note that in a coupled channels problem (e.g. $\pi\pi + K\bar{K}$, recall also the discussion following (2.11)) b may now become a non-constant (diagonal) matrix, leading to several complications in the formalism! Given the present state of the art, it is beyond the scope of this paper to consider such a possibility. It will suffice to stress that the determination of b can be a large source of uncertainty in extracting the P matrix from (2.1).

It is now a simple matter to calculate the S-wave compensation energies from (2.3). Using the standard relativistic relation to convert to energy, and noting that a bag primitive of energy E_0 has a size described by (2.18) we find that E_c is given by the first root above threshold of

$$E_c^4 - 2(m_1^2 + m_2^2)\, E_c^2 + (m_1^2 - m_2^2)^2 - (4\pi^2/c^2)\, (\tfrac{16}{3}\pi B)^{\frac{2}{3}}\, E_c^2 (E_0 - 0.19\, n_s)^{-\frac{2}{3}} = 0. \tag{2.19}$$

A list of some compensation energies normalized to $E_0 = E_c$ is given in table 1.

TABLE 1. S-WAVE COMPENSATION ENERGIES FOR SOME INTERESTING CHANNELS

(For those channels containing s quarks, there is given firstly E_c calculated assuming all quarks to be massless and then E_c with the s quark correction term included.)

configuration	channel	E_c/GeV
$q^2\bar{q}^2$	$\pi\pi$	0.94
	πK	1.10 (1.14)
	$K\bar{K}$, KK	1.28 (1.33)
$q^4\bar{q}$	$N\pi$	1.49
	$\Lambda\pi$	1.63 (1.66)
	$\Sigma\pi$	1.70 (1.72)
	NK, $N\bar{K}$	1.67 (1.69)
	NK^*	1.99 (2.00)
q^6	NN	2.08
	$\Lambda\Lambda$	2.39 (2.40)
	$N\Xi$	2.41 (2.43)

The calculation of the residues presents something of a problem. As Jaffe & Low (1979) have emphasized, only a qualitative answer is possible using the bag model. One has to make additional assumptions about how the bag states couple to the physical channels and how the confinement energy varies as the artificial confinement volume is changed in the presence of a fixed (but unknown) potential. They proceed by noting that the bag eigenstates have projections onto both open (unconfined) and closed (confined and unconfined) channels. Denoting these by ζ_o and ζ_c respectively and assuming that the relative projections do not vary significantly as the confinement radius is varied in the neighbourhood of b implies the physical couplings

$$\xi_o = \zeta_o / \langle\Lambda\rangle^{\frac{1}{2}}, \tag{2.20}$$

where

$$\langle\Lambda\rangle = \sum_o \zeta_o^2. \tag{2.21}$$

Calculation of the residues is completed by determining the derivative $\partial s_0/\partial b$ for which Jaffe & Low (1979) obtained the crude estimate

$$\partial s_0/\partial b = -\tfrac{3}{2}(s_0/b)\, \langle\Lambda\rangle. \tag{2.22}$$

The derivation of (2.22) uses the bag model virial theorem and it should be corrected when s quarks are present. Thence we obtain

$$\partial s_0/\partial b = -\tfrac{3}{2}\{(s_0 - 0.19\, n_s s_0^{\frac{1}{2}})/b\}\, \langle\Lambda\rangle\ \mathrm{GeV}^3. \tag{2.23}$$

Unfortunately both (2.22) and (2.23) are in poor agreement with the data (Jaffe & Low 1979; Roiesnel 1979; Jaffe & Shatz 1980), giving results which are usually too small. Roiesnel (1979) has suggested modifying (2.21) because some confined channels may become open by exchanging a gluon in a first-order process involving higher partial waves. Adding this first-order amplitude incoherently to the 'zero-order' one leads to

$$\langle\Lambda\rangle = \sum_{o} \zeta_{co}^2 + \alpha_c^2 O(1) \sum_{c'} \zeta_{cc'}^2, \tag{2.24}$$

where c′ denotes those closed channels which can become open this way. A prescription of this sort has the benefit of admitting couplings of ordinary baryon and meson primitives to open channels but it should be noted that the neglect of interference effects in (2.24) has little justification; other than a desire to increase $\langle\Lambda\rangle$. Roiesnel (1981, private communication) does note however that if they were taken into account then they would obliterate any information about the residues. Finally on this point, it is remarked that we shall find that Roiesnel's correction to $\langle\Lambda\rangle$ is not generally very successful and the problem seems even more fundamental. Indeed it will be argued elsewhere (Bickerstaff 1983) that the estimates of λ used here should be increased by a factor of $\frac{4}{3}$. However, to facilitate comparison with earlier work we shall not pursue explicitly that aspect further in this paper.

TABLE 2. P-MATRIX POLES AND RESIDUES

(The $q^2\bar{q}^2$ data is from Jaffe & Low (1979); Irving *et al.* (1981), cf. Jaffe (1981); the $q^4\bar{q}$ data is from Roiesnel (1979); Corden *et al.* (1982) and the q^6 data from Jaffe & Shatz (1980), cf. Mulders 1982*b*.)

configuration	channel		pole location/GeV	residue, λ/GeV^3
$q^2\bar{q}^2$	($J^{PC} = 0^{++}$)	$\pi\pi$ (I = 0)	0.69	0.064
		$\pi\pi$, $K\bar{K}$ (I = 0)	1.04, 1.27, 1.47	0.10, —, —
		πK (I = $\frac{1}{2}$)	0.96	0.079
		$\pi\pi$ (I = 2)	1.04	0.21
		πK (I = $\frac{3}{2}$)	1.19	0.22
$q^4\bar{q}$	($J^P = \frac{1}{2}^-$)	$\pi\Sigma$, $\bar{K}N$ (I = 0)	1.45	0.052
		$\pi\Lambda$, $\pi\Sigma$, $\bar{K}N$ (I = 1)	1.54	—
		πN (I = $\frac{1}{2}$)	1.43	0.14
		πN (I = $\frac{3}{2}$)	1.56	—
		KN (I = 0)	1.70 ± 0.01	0.19 ± 0.01
		KN (I = 1)	1.78, 1.95	0.27, —
q^6	$^3S_1$	pn (I = 0)	2.10	0.39 ± 0.02
	$^1S_0$	pn (I = 1)	2.11	0.27 ± 0.05

A compilation of available P-matrix data is contained in table 2. Of course single-channel poles found near to a threshold should be regarded as poles in a reduced P matrix and the actual parameters could be considerably different (Jaffe & Low 1979; Roiesnel 1979). Unfortunately, perhaps, all of the data in table 2 were obtained using b as given via (2.12) rather than the improved version (2.18). Therefore the data for channels with s quarks are subject to greater theoretical uncertainty than those without.

(*b*) *Judging predictions*

To conclude this prelude we must consider the manner of comparing predictions with the data. Even in the absence of a hadron–hadron potential the P matrix will have not just one but numerous poles, at roughly equal spacing. (For example, in the $\pi\pi$ channel there will be com-

pensation poles at 0.94, 1.55, 2.09, ..., GeV). Thus it will prove useful (for the lowest pole in a single channel problem) to define a 'discrepancy' factor

$$\Delta = (\Delta_1 \Delta_2)^{\frac{1}{2}}, \tag{2.25}$$

where

$$\Delta_1 = b\,|E_{\rm pred}\{[E_{\rm obs}^2-(m_1+m_2)^2]\,[E_{\rm obs}^2-(m_1-m_2)^2]\}^{\frac{1}{2}} - E_{\rm obs}\{[E_{\rm pred}^2-(m_1+m_2)^2]\,[E_{\rm pred}^2-(m_1-m_2)^2]\}^{\frac{1}{2}}|/(\pi E_{\rm obs}E_{\rm pred}) \tag{2.26}$$

and

$$\Delta_2 = |E_{\rm pred}-E_{\rm obs}|/(\tfrac{1}{2}E_{\rm obs}). \tag{2.27}$$

Equation (2.26) notes that $(E_{\rm pred}-E_{\rm c})$ is an indication of the scattering phase shift $\delta = b(k_{\rm c}-k_0)$ and attempts to judge the accuracy of the predicted pole position by comparing $|\delta_{\rm pred}-\delta_{\rm obs}|$ with respect to $\frac{1}{2}\pi$. Equation (2.27) determines whether the prediction is close to the observed value in comparison with neighbouring observed poles (which are roughly a distance $E_{\rm obs}$ away). A successful prediction of the pole position will be characterized by a value of Δ much less than unity.

Residue predictions may be compared with data in the normal manner. We note though that the current theory always predicts $r \leqslant \frac{3}{2}s_0/b$ (or $2s_0/b$, Bickerstaff (1983)).

3. Mass calculations

The 'masses' were calculated using the static spherical cavity Hamiltonian (DeGrand *et al.* 1975) but without the colour-electric term, i.e. (2.13) and (2.14). (Neglect of colour-electric effects is expected (DeGrand *et al.* 1975) to lead to errors of the order of 5 MeV in those states with both o and s quarks. This is below the current level of credibility of the model and the omission is in line with earlier work on multiquark states (Jaffe 1977*a*, *b*, Strottman 1979; Mulders *et al.* 1980) with which we wish to make comparison.) The bag parameters used were the same as those found by DeGrand *et al.* (1975) in the case $m_0 = 0$, i.e.

$$\left.\begin{aligned} B^{\frac{1}{4}} &= 0.146\,\text{GeV},\\ Z_0 &= 1.84,\\ \alpha_{\rm c} &= 0.55,\\ m_{\rm s} &= 0.279\,\text{GeV},\end{aligned}\right\} \tag{3.1}$$

and the quark eigenfrequencies and colour-magnetic interaction strengths were computed using the formulae given by DeGrand *et al.* (1975). It was found that the above fit is not sufficiently accurate to enable one to reproduce hadron masses to better than a few MeV but with all the other uncertainties inherent in the model – particularly those relating to centre-of-mass corrections (Donoghue & Johnson 1980; Wong 1981, Carlson & Chachkhunashvili 1981; Liu & Wong 1982; Dethier *et al.* 1983), pionic effects associated with chiral symmetry violations inadvertently imposed by the bag boundary conditions (Chodos & Thorn 1975; Inoue & Maskawa 1975; Brown & Rho 1979; Brown *et al.* 1979; Jaffe 1980, 1982*a*; Miller *et al.* 1981; Thomas *et al.* 1981; Théberge *et al.* 1982; DeTar 1981*a*, *b*), and pair annihilation contributions (DeGrand *et al.* 1975; Jaffe 1977*a*) – there would be little to gain in the present discussion by refining it. Note in this regard that a naïve centre-of-mass correction should not be made when predicting the position of P-matrix poles because the determination of b via (2.9) already accounts

for centre-of-mass motion (Jaffe & Low 1979). (Since the parameters in (3.1) were not obtained by a fit to P-matrix poles but rather by a fit to the masses of the nucleon, Ω^- baryon, and the Δ and ω resonances, one might query whether they are appropriate to the present problem. However, for the four cases just cited the discrepancy between the S-matrix poles and the corresponding P-matrix poles is not expected to be very large. Indeed, for the ρ resonance which is broader than any of these, Jaffe & Low (1979) found that the difference between the two pole positions is only of the order of 10 MeV; see, though, the discussion of the Δ pole by Mulders & Thomas (1982) and Maciel & Paton (1981).) It is nevertheless possible that recoil corrections may enter via single-gluon-exchange because (2.10) assumes non-interacting quark eigenstates. Chiral effects may also be important (Thomas 1982; Mulders & Thomas 1982) and both points deserve study. However, we shall neglect both of these in this survey; one of the primary purposes of which is to investigate the ramifications and consequences of removing Jaffe's approximation to the colour-hyperfine spectra, and these effects would cloud the issue.

The actual calculations were performed in a straightforward manner by combining the matrix elements of the colour-spin operator, Δ^{ab}, with the appropriate radial integrals M_{ab} at the bag radius, R, and diagonalizing the resulting matrix of the colour-hyperfine term. The eigenvalues were then combined with the remaining volume, 'zero-point' and quark kinetic energy terms, the procedure repeated for various values of R and each eigenvalue of H minimized separately. When only massless quarks are present the minimum energy and radius can easily be determined analytically (with (2.16)) but for those states containing s quarks the E against R curve was fitted using a cubic plus $1/R$ term. This is very accurate. The procedure has the advantage that the mixing induced by the flavour dependence of the M_{ab} can be traced as a function of R. For small values of R, M_{ab} tends to M_{00} and drops outside the summation over flavour pairs. Thus the eigenvectors at small R approach those in the Jaffe limit.

In general each eigenenergy passes through a minimum at a different value of the bag radius and this results in the eigenvectors not being exactly orthogonal. It is remarked that even within a given matrix with the same number of massive quarks the tendency for the bag radius to increase with increasing eigenenergy is not precise.

As might be gathered from the above, considerable care was taken to ensure that round-off errors did not affect the answers. Nevertheless it should be clear from the earlier remarks that it is only of academic interest to quote energies to the nearest MeV. When we do so it is only to give an accurate picture of the effects of removing the Jaffe approximation. The absolute values are not that reliable because of limitations in the model.

4. $q^2\bar{q}^2$ primitives

The matrix elements of the colour-spin operator in the $q^2\bar{q}^2$ configuration can be found tabulated in (Bickerstaff 1980). All of the matrices have been checked by comparing their eigenvalues with known eigenvalues in both the Jaffe (1977*a*, *b*) limit and the limit in which quarks and antiquarks are all treated on the same footing (Bickerstaff 1980; Bickerstaff & Wybourne 1980). Since the flavour dependence of the colour-hyperfine term only enters through the radial integrals, M_{ab}, it suffices to calculate the colour-spin operator within a generic flavour configuration (Bickerstaff & Wybourne 1980). The colour-spin operator will mix states having the same internal SU_6^{CJ} quantum numbers and the same total spin and these suffice to determine the mixing matrices. We shall follow Bickerstaff (1980) in labelling these matrices by a capital script

letter, arbitrarily associated with the internal SU_6^{CJ} quantum numbers. The spin multiplicity will be denoted by a left superscript. These labels are defined in table 3. Thus the largest matrix is seen to be the six dimensional ${}^3\mathscr{F}$.

TABLE 3. $q^2\bar{q}^2$ COLOUR–SPIN MATRICES

(The SU_6^{CJ} quantum numbers are denoted by Schur functions (essentially Young tableaux) and refer to the quark subsystems in the generic flavour configuration.)

matrix name	generic configuration	internal SU_6^{CJ} quantum numbers	dimension in $J = 0$	dimension in $J = 1$	dimension in $J = 2$
$\mathscr{A}$	$q_a^2\bar{q}_b^2$	$\{2\}\{2^5\}$	2	1	1
$\mathscr{B}$	—	$\{1^2\}\{1^4\}$	2	1	1
$\mathscr{C}$	—	$\{2\}\{1^4\}$	—	2	—
$\bar{\mathscr{C}}$	—	$\{1^2\}\{2^5\}$	—	2	—
$\mathscr{D}$	$q_a q_b \bar{q}_c^2$	$\{1\}\{1\}\{2^5\}$	2	3	1
$\mathscr{E}$	—	$\{1\}\{1\}\{1^4\}$	2	3	1
$\bar{\mathscr{D}}$	$q_a^2\bar{q}_b\bar{q}_c$	$\{2\}\{1^5\}\{1^5\}$	2	3	1
$\bar{\mathscr{E}}$	—	$\{1^2\}\{1^5\}\{1^5\}$	2	3	1
$\mathscr{F}$	$q_a q_b \bar{q}_c \bar{q}_d$	$\{1\}\{1\}\{1^5\}\{1^5\}$	4	6	2

In Jaffe's approximation the largest matrices are 2×2. They may be identified by the flavour multiplets associated with the eigenstates, together with the total spin. We shall follow Jaffe (1977*a*, *b*) in using dimensional notation to refer to the (reducible) SU_3^{fl} representations. In terms of S-functions (or Young tableaux) those arising are $\underline{9} \equiv \{0\}+\{21\}$, $\underline{18} \equiv \{21\}+\{3\}$, $\overline{\underline{18}} \equiv \{21\}+\{3^2\}$ and $\underline{36} \equiv \{0\}+\{21\}+\{42\}$. Where Jaffe's matrices are two-dimensional the eigenstate with the highest energy is distinguished by an asterisk. Thus $\underline{9}^*$ denotes a colour-spin excitation of the flavour nonet and is not to be confused with a separate flavour representation.

Removing Jaffe's approximation may result in mixing between these matrices or it may just modify the eigenvectors in the 2×2 matrices. A comparison of Jaffe's (1977*a*) energy levels with the current results is shown in figure 1. As can be seen, there is no drastic modification of the eigenenergies. Note that Jaffe's results are rounded to the nearest 50 MeV so that the energy shifts cannot be taken literally. In fact in some cases there should be no shift at all.

(*a*) $J^P = 0^+$

The only states of immediate physical interest are the low-lying 0^+ primitives. Let us consider exotic channels first. The ${}^1\mathscr{B}$ state in the $(I, S) = (2, 0)$ channel at 1122 MeV is a pure $o^2\bar{o}^2$ state and thus Jaffe's approximation is exact in this case. One finds that the state comprises 41.5 % $\pi\pi$ components, 3.1 % $\rho\rho$, 16.6 % $\boldsymbol{\pi}\cdot\boldsymbol{\pi}$ and 38.8 % $\boldsymbol{\rho}\cdot\boldsymbol{\rho}$, (i.e. 55.4 % colour octet–octet combinations). Since it is above the $\pi\pi$ threshold (but well below the $\rho\rho$ threshold) the open channel projection is $\xi_{\pi\pi} = 1$ and we expect to see a pole in the $\pi\pi(I = 2)$ P matrix at 1.12 GeV, above the compensation energy at 0.94 GeV signalling a negative phase shift. From (2.23), the predicted residue is 0.11 GeV^3. Because there are no s quarks present, the bag radius is correctly predicted by the virial theorem to be $m_s R_0 = 1.474$ and we ought to be entitled to expect reasonable agreement with the data (in table 2). As noted by Jaffe & Low (1979) the phase shift is indeed negative – and falling and definitely non-resonant – and there is fair agreement with the predicted pole position, the discrepancy being $\Delta = 0.17$. However the predicted residue is in error by nearly 100 %. Roiesnel's correction only boosts the prediction to 0.12 GeV^3; still well short of the observed value.

The ${}^1\bar{\mathscr{E}}$ state at 1322 MeV in the $(\frac{3}{2}, 1)$ channel is an $o^2\bar{o}\bar{s}$ state and therefore one would expect

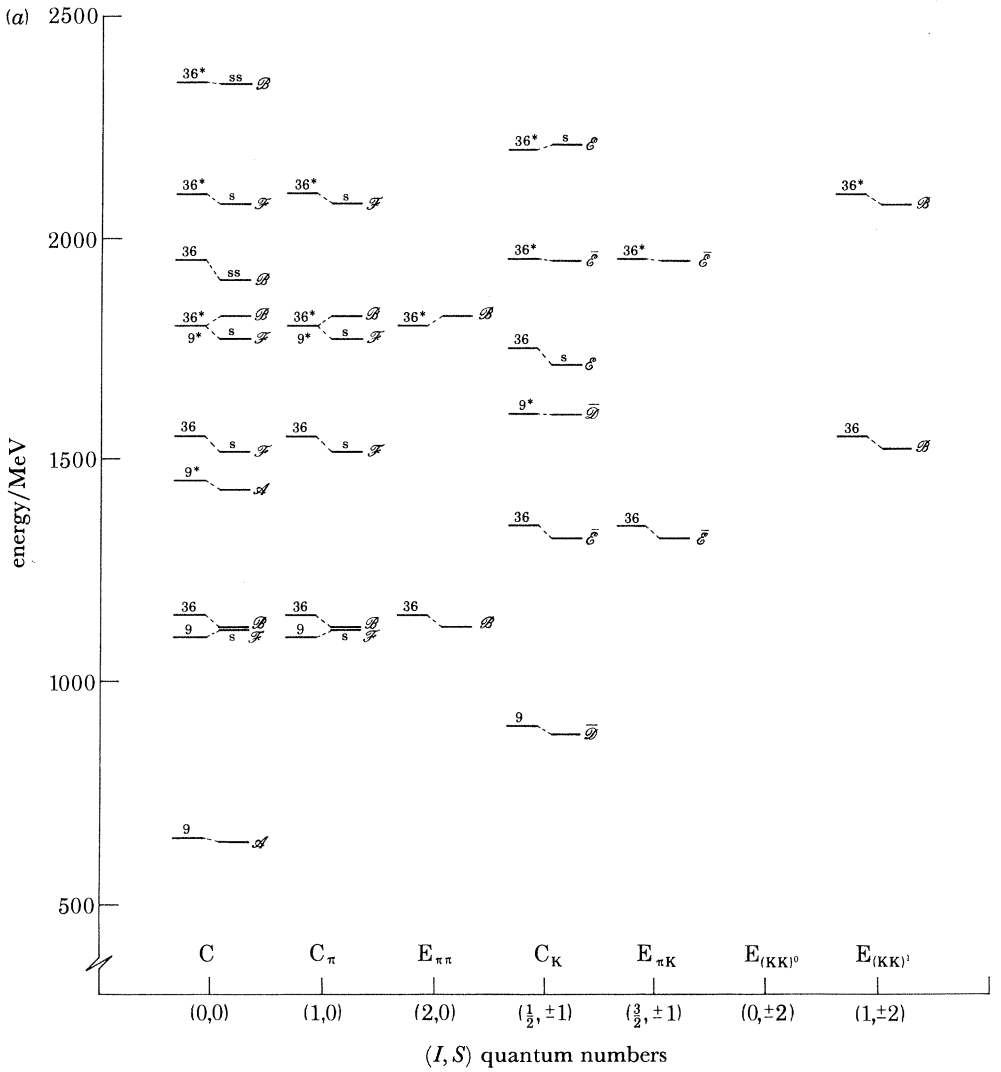

FIGURE 1. $q^2\bar{q}^2$ S-wave primitives. The channels are labelled both by their isospin and strangeness quantum numbers and Jaffe's (1977*a*, *b*) notation which depicts whether it is exotic (E) or cryptoexotic (C). For each channel the results on the left are those of Jaffe (1977*a*) (rounded to the nearest 50 MeV) while those on the right are the current results, which incorporate the effects of mixing induced by the flavour dependence of the colour magnetic interaction strengths. Jaffe's states are labelled with their SU_3^f quantum numbers in his dimensional notation while the improved states display the mixing matrix from which they derive. The presence of a hidden $s\bar{s}$ pair is indicated with the letter s, which enables the specific flavour content of the state to be determined.

(*a*) $J^P = 0^+$. The matrix labels on the $S = \pm 1$ states only refer to $S = +1$. The $S = -1$ states come from the barred matrices.

(*b*) $J^P = 1^+$. The matrix labels and the flavour multiplets $\underline{18}$ and $\overline{\underline{18}}$ in the channels $(\frac{1}{2}, \pm 1), (\frac{3}{2}, \pm 1), (0, \pm 2)$ refer to the positive S states. For the negative S states these labels must be barred.

(*c*) $J^P = 2^+$. The matrix labels on the $S = \pm 1$ states refer to the $S = +1$ states. The $S = -1$ states come from the barred matrices.

mixing to occur. However, the ${}^1\bar{\mathscr{E}}$ matrix is such that (for this specific configuration) the radial integrals can be extracted in the combination $M_{00} + M_{0s}$ from the summation in the colour-hyperfine term and thus the state is pure $\underline{36}$ in flavour. Hence, in this case the validity of the Jaffe approximation is solely a question of the accuracy of the averaging procedure used for the interaction strengths. The present calculation results in the energy already quoted and the dimension-

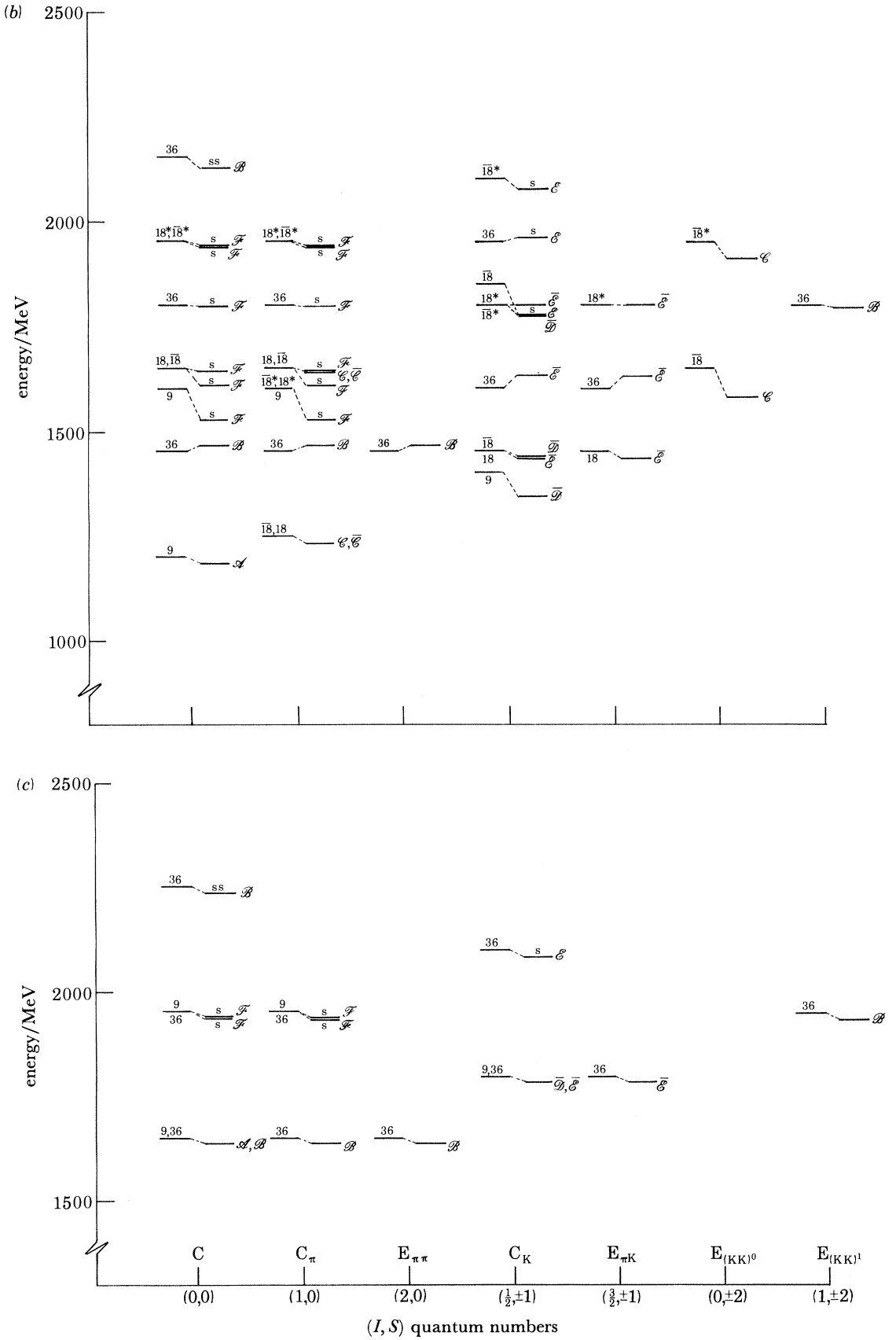

FIGURE 1 (*b*, *c*). For caption, see opposite.

less bag radius $m_s R_0 = 1.463$. We find the meson–meson content to be 41.5 % πK, 3.1 % ρK*, 16.6 % $\boldsymbol{\pi}\cdot\mathbf{K}$ and 38.8 % $\boldsymbol{\rho}\cdot\mathbf{K}^*$ in agreement with expectations for a pure $\underline{36}$ state (Wong & Liu 1980). The open channel projection is $\xi_{\pi K} = 1$ and we expect to see a pole in the $\pi K(I = \frac{3}{2})$ P matrix at 1.32 GeV, above $E_c(\pi K) = 1.14$ signalling a negative phase shift. The predicted residue is 0.13 GeV^3; a little less than that predicted by Jaffe & Low (1979) because of the s quark mass correction. There is qualitative agreement with experiment where the phase shift is indeed negative but the quantitative agreement is only fair, the discrepancy in the pole position being $\Delta = 0.28$ and the residue being in error by 70 %. Roiesnel's correction only boosts the predicted residue to 0.14 GeV^3. The discrepancy in the pole position is not entirely unexpected because the values in table 2 were deduced assuming massless quarks. This results in a value for b which is too large. Because the residue is fairly big the pole position is sensitive to the value of b. Using a smaller value of b would raise the experimental point towards the predicted value.

The $^1\mathscr{B}$ state at 1524 MeV in the (1, 2) channel belongs to the $o^2\bar{s}^2$ specific configuration. Although some mixing does occur between the $\underline{36}$ and $\underline{36}^*$ states it is negligibly small and this state is therefore almost pure $\underline{36}$. As no data exist for this channel we shall move onto the crypto-exotic primitives.

In the (0, 0) channel a cryptoexotic $^1\mathscr{A}$ state is found at only 642 MeV. Since all quarks in this state are ordinary, Jaffe's approximation is exact and the state is pure flavour $\underline{9}$. The primitive is found to be 41.4 % $\pi\pi$, 13.8 % $\eta_0\eta_0$, a very small amount of $\omega_0\omega_0$, 0.1 % $\rho\rho$ and 44.6 % colour octet components. The η_0 and ω_0 mesons are pure $o\bar{o}$ states. Their presence here poses a problem because it suggests the necessity for pair annihilation interactions which have been neglected in this calculation. These will mix this state with the other states possessing the same quantum numbers (Bickerstaff & McKellar 1982). (Note that simply writing η_0 as a linear combination of the physical η and η' does not take account of this mixing.) Ignoring this problem for the present we observe that this primitive lies below the $\eta\eta$ threshold and thus the open channel coupling is $\xi_{\pi\pi} = 1$. The predicted residue is 0.042 GeV^3 in agreement with Jaffe & Low (1979). (Roiesnel's correction makes little difference.) Since the primitive is below $E_c(\pi\pi) = 0.94$ GeV the phase shift is (correctly) predicted to be positive. The agreement with data is quite reasonable, with $\Delta = 0.13$, but again the predicted residue is too small. Although the P-matrix pole is extracted from that part of the phase shift associated with the dubious $\varepsilon(700)$ there is no reason for presuming that this primitive has any close relation with a resonance.

Also in this channel is a $^1\mathscr{F}$ state at 1115 MeV. Jaffe & Low (1979) have associated this state with the P-matrix pole at 1.04 GeV which they argue is connected with the S*(980) because the $K\bar{K}$ threshold induces a pole near 0.98 GeV in the effective $\pi\pi$ P matrix having a rather small residue. However, in addition to this primitive there is a $^1\mathscr{B}$ state only 7 MeV higher at 1122 MeV. Even the most optimistic bag enthusiast must regard this difference as being of the order of accuracy of the model. (Note for example that including a colour-electric contribution would raise the $^1\mathscr{F}$ state by about this amount.) Thus we need to consider both of these primitives in connection with the pole at 1.04 GeV. The $^1\mathscr{F}$ states contain an $s\bar{s}$ pair and exhibit some mixing between the $\underline{9}$, $\underline{36}$, $\underline{9}^*$ and $\underline{36}^*$ C^s states of Jaffe (1977*a*, *b*). This mixing is seen as a radial dependence of the $^1\mathscr{F}$ eigenstates. However the actual variation, and hence the mixing, is not very large. Thus the lowest $^1\mathscr{F}$ state is almost a pure flavour $\underline{9}$. The $^1\mathscr{B}$ states are all $o^2\bar{o}^2$ and therefore Jaffe's approximation is exact and the lowest $^1\mathscr{B}$ state is pure flavour $\underline{36}$. The $^1\mathscr{F}$ state is comprised of 27.0 % $K\bar{K}$, 28.2 % $\eta_0\eta_s$, 0.1 % $\omega_0\phi_s$, 0.1 % $K^*\bar{K}^*$ and 44.6 % colour octet components. In contrast the $^1\mathscr{B}$ state comprises 10.4 % $\pi\pi$, 31.1 % $\eta_0\eta_0$, 2.3 % $\omega_0\omega_0$, 0.8 % $\rho\rho$ and 55.4 %

colour octet components. It is clear that (possibly substantial) pair annihilation mixing will occur between these states and thus neither their energies nor their meson–meson contents can be regarded as very reliable predictions. Indeed it can be shown (Bickerstaff & McKellar 1982) that the nonlinear nature of this mixing is capable of drastic modifications. It is of some interest here to note that the $s\bar{s}$ content of the state at 1115 MeV results in $m_s R_0 = 1.200$. The value predicted by the virial theorem is too large by 23 %! In constrast, use of (2.18) gives an answer in error by less than 7 %. (This is the most disagreeable case encountered among all the $q^2\bar{q}^2$, $q^4\bar{q}$ and q^6 primitives.) This large difference in size between these two primitives – the other of course having $m_s R_0 = 1.474$ according to the virial theorem – is a crucial factor in producing the curious effects found by Bickerstaff & McKellar (1982). We remark here merely that a surprisingly large $\pi\pi$ component was found to be possible for the lowest mixed state. The reader is referred to that paper for a discussion of the open channel couplings $\xi_{\pi\pi}$ and $\xi_{K\bar{K}}$ which were found to be rather sensitive to the details of mixing. Obviously this applies also to the residues, although to a somewhat lesser degree. (For the record, the prediction for λ lies in the range 0.03–0.06 GeV^3; again less than the observed value. Indeed in Bickerstaff & McKellar (1982) the lower extreme was favoured.)

Further poles have been found in this channel but we shall not attempt to compare them with $q^2\bar{q}^2$ bag predictions as we also expect a $q\bar{q}$ primitive associated with the $\varepsilon(1300–1400)$. Better data seem to be necessary to assist with the correct identification. At present it appears that there may be insufficient poles to accommodate both the $\varepsilon(1300–1400)$ and the $q^2\bar{q}^2$ states. We shall be content to note that several nearby primitives are theoretically possible because of the number of open channels (Low 1982).

The pair of primitives at 1115 and 1122 MeV is degenerate with another pair in the (1, 0) channel and the primitive at 1122 MeV is further degenerate with the exotic $I = 2$ $^1\mathscr{B}$ state already discussed. (Many other cases of such degeneracies can be seen in figure 1.) This isospin degeneracy is of the same nature as the $\pi\eta_0$ and $\rho\omega_0$ degeneracies encountered in the bag model treatment of ordinary mesons (DeGrand *et al.* 1975) and stems from the independence of the colour-hyperfine interaction on the total isospin. Pair annihilation contributions will though provide different mixing in each of the isospin channels and will break this degeneracy; in a similar fashion to the $\pi\eta_0$ and $\rho\omega_0$ degeneracy breaking. In the absence of mixing the $I = 1$ $^1\mathscr{F}$ state is 27.0 % $K\bar{K}$, 28.2 % $\pi\eta_s$, 0.1 % $\rho\phi_s$, 0.1 % $K^*\bar{K}^*$ and 44.6 % colour octet components. The $^1\mathscr{B}$ state is 41.5 % $\eta_0\pi$, 3.1 % $\omega_0\rho$ and 55.4 % colour octet components. Note that it does not couple to either $\pi\pi$ or $\rho\rho$ channels because of G-parity conservation. (This is reflected in the dissociation calculation through an isospin $9j$ symbol having the value zero by permutation symmetry.) Mixing between these states is discussed by Bickerstaff & McKellar (1982). Unfortunately there is no $\pi\eta$ data with which to perform a P-matrix analysis for this channel. However we expect to see a pole in the effective $\pi\eta$ P matrix, roughly degenerate with the $I = 0$ $\pi\pi$ pole near 0.98 GeV and having an artificially small residue. This suggests (Jaffe & Low 1979) that these primitives might be associated with the $\delta(980)$.

The lowest state in the cryptoexotic $(\frac{1}{2}, 1)$ channel is a $^1\bar{\mathscr{D}}\,o^2\bar{o}s$ state. Although we might have expected mixing to take place when the Jaffe approximation is removed the $^1\bar{\mathscr{D}}$ matrix is of such a form that the combination $M_{oo} + M_{os}$ can be extracted from the summation in the colour-hyperfine term, in an analogous manner to the situation encountered for the $^1\bar{\mathscr{E}}$ matrix. Thus this state is a pure flavour nonet. Its energy is found to be 882 MeV with $m_s R_0 = 1.209$. It comprises 41.4 % πK, 13.8 % $\eta_0 K$, 0.1 % ρK^*, a very small amount of $\omega_0 K^*$, 2.1 % $\boldsymbol{\pi}\cdot\mathbf{K}$, 0.7 % $\boldsymbol{\eta}_0\cdot\mathbf{K}$,

31.3 % $\rho \cdot K^*$ and 10.4 % $\omega_0 \cdot K^*$. Thus the open channel projection is $\xi_{\pi K} = 1$ and we expect to see a primitive in the πK P matrix at 0.88 GeV below the compensation energy and therefore the phase shift should be positive. The predicted residue is 0.062 GeV^3 which is slightly modified by Roiesnel's correction to be 0.063 GeV^3. (Notice that the s quark mass correction gives a substantial reduction from the Jaffe–Low result.) Agreement with the data is fair. The discrepancy in the pole position is $\Delta = 0.19$ but the residue is too small. Further, the agreement may be worse than it at first seems because the pole position in table 2 was determined using a value of b which is too large. Reducing b will raise the pole position and increase the discrepancy. We hope that the rather small residue will restrain this shift to within tolerable limits. As noted by Jaffe & Low (1979) this primitive is associated with the broad enhancement known as the κ(800–1100) but the P-matrix interpretation nicely avoids any necessity to interpret this as a resonant bag state.

Thus we have seen that removing Jaffe's approximation makes either very little or no difference to the 0^+ states. Apart from some changes in the residues due to the present inclusion of a correction for the s quark mass the results we have obtained here are essentially those of Jaffe & Low (1979). This is in spite of the fact that those authors used results from (Jaffe 1977*a*, *b*) which, it is now realized, contains phase inconsistencies (Wong & Liu 1980). However, the open channels considered here contain only pseudoscalar mesons and fortunately Jaffe's (1977*a*) couplings to those channels are correct (Wong & Liu 1980). Use of his other couplings would give erroneous results for Roiesnel's correction to the residues and in pair annihilation mixing calculations (Bickerstaff & McKellar 1982).

(*b*) $J^P = 1^+$

No P-matrix data exist to compare with the 1^+ and 2^+ primitives. However these states display some interesting examples of mixing which we shall discuss. First, consider the 1^+ primitives. The $^3\mathscr{A}$ and $^3\mathscr{B}$ matrices are one-dimensional and therefore their eigenstates are pure $\underline{9}$ and $\underline{36}$ in flavour respectively. Indeed Jaffe's approximation is exact in the $o^2\bar{o}^2$ and $s^2\bar{s}^2$ configurations and even for the o^2s^2 $^3\mathscr{B}$ state where it predicts a null colour-hyperfine contribution, the true contribution is only 2 MeV. In the (1, 0) channel, the $^3\mathscr{C}$ and $^3\overline{\mathscr{C}}$ matrices are also exactly described by Jaffe's approximation and their eigenvectors are pure $\underline{\overline{18}}$ or $\underline{\overline{18}}^*$ and $\underline{18}$ or $\underline{18}^*$ flavour states respectively. Thus these states remain degenerate. Note however that they are not eigenstates of G-parity and one must take the symmetric and antisymmetric combinations $C_\pi^\pm = (1/2)^{\frac{1}{2}}\{C_\pi(\underline{18}) \mp C_\pi(\underline{\overline{18}})\}$ as discussed by Jaffe (1977*a*). In the (0, 2) channel the $^3\mathscr{C}$ matrix does exhibit some mixing but it is very small. The $^3\mathscr{D}$ and $^3\overline{\mathscr{E}}$ matrices contain interesting examples of mixing. In the Jaffe approximation these 3×3 matrices block-diagonalize into 2×2 and 1×1 matrices and the 2×2 matrices are the same for $^3\overline{\mathscr{D}}$ and $^3\overline{\mathscr{E}}$. This gives rise to the $\underline{18}$, $\underline{\overline{18}}$ and $\underline{18}^*$, $\underline{\overline{18}}^*$ degeneracies in figure 1*b*. (Note that Jaffe's (1977*a*) tabulation of these states inadvertently omits a $C_{\bar{K}}$ state in the $\underline{18}$ and $\underline{18}^*$ at 1450 and 1800 MeV respectively. Their antiparticles in the $\underline{\overline{18}}$ and $\underline{\overline{18}}^*$ provide C_K states degenerate with the C_K states which Jaffe does list and it is this degeneracy which can be seen in figure 1*b*.) Because the third state with which these states mix is different in the two matrices, the $\underline{18}$, $\underline{\overline{18}}$ and $\underline{18}^*$, $\underline{\overline{18}}^*$ degeneracies are raised. Indeed the proximity of the unmixed $C_K(\underline{9})$ and C_K $(\underline{\overline{18}})$ leads to very strong mixing in the $^3\overline{\mathscr{D}}$ matrix. The lowest $^3\overline{\mathscr{D}}$ state at 1344 MeV is only 64 % flavour $\underline{9}$ with the rest being almost entirely $\underline{\overline{18}}$. Correspondingly, the next eigenvector at 1437 MeV is only $\underline{64}$ % $\underline{\overline{18}}$ with the rest being mostly $\underline{9}$. The highest eigenstate at 1773 MeV remains largely unaffected and is almost pure $\underline{\overline{18}}^*$. In contrast the mixing within $^3\overline{\mathscr{E}}$ is rather moderate. The eigenstates at 1433, 1631 and 1801 MeV are 98 %, 97 % and 99 % pure respectively with the strongest mixing clearly being between the $\underline{18}$ and $\underline{36}$ states. In the $^3\overline{\mathscr{E}}$

matrix, mixing is similarly moderate at about the 1–2 % level. Mixing is also strong in the $^3\mathscr{F}$ matrix. In the Jaffe limit it block diagonalizes into two degenerate 2×2 matrices and two (non-degenerate) 1×1 matrices. Mixing raises the degeneracy of the $\underline{18}$, $\overline{\underline{18}}$ and $\underline{18}^*$, $\overline{\underline{18}}^*$ states. The six eigenenergies found are at 1526, 1607, 1642, 1797, 1934 and 1941 MeV, so we see that the splitting of the $\underline{18}$, $\overline{\underline{18}}$ pair is fairly substantial. All eigenvectors are correctly found to be eigenstates of G-parity (which provides an additional check on the mixing matrix). Their 'C-parities' are $-, +, -, -, +, -$ respectively (i.e. four eigenvectors change sign under conjugation of their colour and spin content and two do not). Examining their flavour content we find that the lowest eigenstate is only 64 % $\underline{9}$ with about 35 % $\underline{18}$ & $\overline{\underline{18}}$ and 1 % $\underline{36}$. Only the $\underline{18}$ & $\overline{\underline{18}}$ state with the same parity mixes. The $\underline{18}$ & $\overline{\underline{18}}$ and $\underline{18}^*$ & $\overline{\underline{18}}^*$ states with positive 'C-parity' mix only amongst themselves and then only to a fairly small degree. The eigenstate at 1642 MeV is 62 % $\underline{18}$ & $\overline{\underline{18}}$, 36 % $\underline{9}$ and 3 % $\underline{36}$ while the next state is 94 % $\underline{36}$ and about 4 % $\underline{18}$ & $\overline{\underline{18}}$ and 2 % $\underline{18}^*$ & $\overline{\underline{18}}^*$. The highest energy eigenstate at 1941 MeV is 98% $\underline{18}^*$ & $\overline{\underline{18}}^*$ with the rest being mostly $\underline{36}$. Thus we see that removing Jaffe's approximation does lead to some quite substantial changes in some of the 1^+ $q^2\bar{q}^2$ wave functions even though the changes in energy are not too great.

(c) $J^P = 2^+$

Within the 2^+ primitives, Jaffe's approximation provides a fair description for all except the $^5\mathscr{F}$ matrix. Indeed the degeneracy of the $^5\mathscr{A}$ and $^5\mathscr{B}$ matrices in the $(0, 0)$ channel is exact, as is the degeneracy of the $^5\overline{\mathscr{D}}$ and $^5\overline{\mathscr{E}}$ matrices in the $(\frac{1}{2}, 1)$ channel. The only point to be wary of is the validity of Jaffe's averaging procedure. Consider then the $^5\mathscr{F}$ matrix. In the Jaffe limit it is diagonal and the eigenvalues are degenerate. It should be noted therefore that the eigenvectors are then arbitrary and the coupling to meson–meson channels is not predictable under such circumstances! If we select those states which are pure $\underline{9}$ and $\underline{36}$ in flavour as basis states, then removing Jaffe's approximation leads to an off-diagonal colour-hyperfine matrix element proportional to $(M_{oo} - 2M_{os} + M_{ss})$ which mixes the flavour states. The full colour-magnetic matrix is in fact (Bickerstaff 1980)

$$\begin{bmatrix} \frac{10}{3}M_{oo} + \frac{12}{3}M_{os} + \frac{10}{3}M_{ss} & 2\sqrt{2}M_{oo} - 4\sqrt{2}M_{os} + 2\sqrt{2}M_{ss} \\ 2\sqrt{2}M_{oo} - 4\sqrt{2}M_{os} + 2\sqrt{2}M_{ss} & \frac{4}{3}M_{oo} + \frac{24}{3}M_{os} + \frac{4}{3}M_{ss} \end{bmatrix},$$

and one readily finds that its eigenvalues are $-\frac{2}{3}M_{oo} + 12M_{os} - \frac{2}{3}M_{ss}$ and $\frac{16}{3}M_{oo} + \frac{16}{3}M_{ss}$ with eigenvectors $\sqrt{\frac{1}{3}}\,\underline{9} - \sqrt{\frac{2}{3}}\,\underline{36}$ and $\sqrt{\frac{2}{3}}\,\underline{9} + \sqrt{\frac{1}{3}}\,\underline{36}$ respectively. Thus mixing raises the degeneracy by 6 MeV but furthermore the eigenvectors are seen to be independent of the magnitude of the off-diagonal element (i.e. no radial dependence of these eigenvectors is observed) and we must therefore conclude that even in the Jaffe limit these are the correct eigenvectors to choose. Hence one obtains the following rather startling results. The lower eigenstate at 1932 MeV and $m_s R_0 = 1.652$ has zero coupling to unconfined channels and in the $I = 0$ case is in fact 50 % $\mathbf{K}^* \cdot \overline{\mathbf{K}}^*$ and 50 % $\boldsymbol{\omega}_0 \cdot \boldsymbol{\phi}_s$, which implies that it could be resonant. The upper eigenstate at 1938 MeV and $m_s R_0 = 1.651$ – note that it is slightly smaller – is the reverse of this with 50 % $\mathrm{K}^*\overline{\mathrm{K}}^*$, 50 % $\omega_0\phi_s$ and no confined components. The only hope of detecting its presence would be by looking for its associated P-matrix pole. The $I = 1$ states are the same but with ρ substituted for ω_0. Of course other sources of mixing could, in principle, swamp these results. It is of particular interest to consider the consequences of first-order pair annihilations (Bickerstaff & Joshi 1982). Whatever actually happens, this example highlights the unreliability of Jaffe's (1977*a*) eigenvectors. It is also worth noting that this extreme example of mixing cannot occur in $q^4\bar{q}$ or q^6

because all states there have the same colour content, irrespective of flavour and spin (Bickerstaff *et al.* 1982*b*) and it is clear from that work that this result is immune to mixing.

Considering the large amount of mixing between the $\underline{9}$ and $\underline{36}$ flavour states in the ${}^5\mathscr{F}$ matrix it should be clear that no predictions are possible for the content of the $(0,0)$ and $(\frac{1}{2},1)$ states which are still degenerate. This degeneracy problem has been overlooked in some recent attempts (Li & Liu 1982; Achasov *et al.* 1982) to relate the $(0,0)$ and $(2,0)$ primitives to structure seen in the two photon production of hadrons. Thus, not only can one question (Jaffe 1981) the physical assumptions underlying those calculations but also one must be very wary of their numerical predictions which are based on pure $\underline{9}$ and $\underline{36}$ flavour states.

TABLE 4. $q^4\bar{q}$ COLOUR–SPIN MATRICES

(The SU_6^{CJ} quantum numbers of the quark subsystems in the generic configuration are denoted by Schur functions. Beside the dimensions (and in parentheses) are shown the arbitrary labels attached to each of these matrices in (Bickerstaff & Wybourne 1980).)

matrix name	generic configuration	internal SU_6^{CJ} quantum numbers	dimension in $J=\frac{1}{2}$	dimension in $J=\frac{3}{2}$	dimension in $J=\frac{5}{2}$
$\mathscr{A}$	$q_a^4\bar{q}_b$	$\{21^2\}\{1^5\}$	2 (a)	2 (b)	1 (c)
$\mathscr{B}$	—	$\{2^2\}\{1^5\}$	1 (d)	1 (e)	—
$\mathscr{C}$	—	$\{1^4\}\{1^5\}$	1 (f)	1 (g)	—
$\mathscr{D}$	$q_a^3q_b\bar{q}_c$	$\{21\}\{1\}\{1^5\}$	5 (a)	4 (b)	1 (c)
$\mathscr{E}$	—	$\{1^3\}\{1\}\{1^5\}$	3 (d)	3 (e)	1 (f)
$\mathscr{F}$	$q_a^2q_b^2\bar{q}_c$	$\{2\}\{1^2\}\{1^5\}$	4 (g)	3 (h)	1 (i)
$\mathscr{G}$	—	$\{1^2\}\{1^2\}\{1^5\}$	4 (k)	4 (l)	1 (m)

5. $q^4\bar{q}$ PRIMITIVES

The matrix elements of the colour-spin operator have been tabulated by Bickerstaff (1980) and Bickerstaff & Wybourne (1980). A summary of the matrices arising for each generic configuration is given in table 4. Again the matrices can be checked in both the Jaffe limit and the limit in which quarks and antiquarks are all treated on the same footing. In using these matrices one should note that the states in the configuration $q_a q_b^3 \bar{q}_c$ are not identical to states in $q_b^3 q_a \bar{q}_c$ because permutation symmetry may introduce a sign change. However the easiest way to treat this case is to diagonalize the matrices as for the $q_b^3 q_a \bar{q}_c$ specific configuration and then to change signs of certain components in the eigenvectors as required (Bickerstaff 1980). The details are straightforward (and uninteresting) but this must be done or else the channel couplings will turn out incorrectly.

This time the largest matrix is the five dimensional ${}^2\mathscr{D}$. If we perform a basis transformation so that the q^4 subsystem has good SU_6^{CJ} symmetry then the matrices in Jaffe's approximation block-diagonalize into 2×2 and 1×1 matrices; as found by Strottman (1979). Once more it is possible to identify the (2×2 and 1×1) matrices in the Jaffe limit by the flavour multiplets associated with the eigenstates, together with the total spin. We shall follow Roiesnel's (1979) dimensional notation for the (reducible) SU_3^{fl} representations, i.e. $\underline{9}\equiv\{0\}+\{21\}$, $\overline{\underline{18}}\equiv\{21\}+\{3^2\}$, $\underline{45}\equiv\{21\}+\{3\}+\{42\}$ and $\underline{45}'\equiv\{3\}+\{51\}$. However, we shall adhere to Jaffe's use of an asterisk to denote colour-spin excitations of these multiplets rather than using Roiesnel's subscripts A and B. It is useful to note that there is a one-to-one correspondence between these flavour multiplets and the SU_6^{CJ} quantum numbers of the q^4 subsystem which were employed for matrix identification by Strottman (1979). The correspondence is: $\underline{9}\leftrightarrow\{31\}^{CJ}$; $\overline{\underline{18}}\leftrightarrow\{2^2\}^{CJ}$; $\underline{45}\leftrightarrow\{21^2\}^{CJ}$ and $\underline{45}'\leftrightarrow\{1^4\}^{CJ}$.

It is worth recording the actual eigenvalues of the combined colour-spin operator in this approximation. If the group generators in equation (1.1) have Jaffe's (1977*b*) normalization then we have the following eigenvalues, in increasing order. For the $\frac{1}{2}^-$ states there are $\frac{8}{3}(-8-\sqrt{31})$, $-\frac{56}{3}$, $\frac{8}{3}(-8+\sqrt{31})$, $\frac{8}{3}(1-\sqrt{10})$, $\frac{8}{3}(1+\sqrt{10})$, $\frac{88}{3}$ corresponding to $\underline{9}$, $\overline{\underline{18}}$, $\underline{9}^*$, $\underline{45}$, $\underline{45}^*$ and $\underline{45}'$ respectively; for the $\frac{3}{2}^-$ states there are -12, $-\frac{20}{3}$, $+\frac{4}{3}$, $+\frac{40}{3}$, $+\frac{40}{3}$ corresponding to $\underline{45}$, $\underline{9}$, $\overline{\underline{18}}$, $\underline{45}^*$ and $\underline{45}'$ respectively; and for the $\frac{5}{2}^-$ states there is just $+\frac{40}{3}$ corresponding to $\underline{45}$. Note the degeneracy of the $\underline{45}^*$ and $\underline{45}'$ $\frac{3}{2}^-$ states.

Recently, Wroldsen (1981) has produced a tabulation of the colour-magnetic matrices in a basis where the q^4 subsystem has good SU_6^{CJ} symmetry. His matrices were computer-generated using first principles and provide an interesting comparison with those in Bickerstaff (1980) and Bickerstaff & Wybourne (1980). His tabulated eigenvectors appear to be correct but unfortunately his discussion of mixing in that preliminary report is not and moreover his treatment of the flavour-symmetric limit is oversimplified and misleading. Consequently his conclusions are at variance with those made here.

A comparison of Strottman's (1979) energy levels with the current ones is presented in figure 2. Because Strottman's results are rounded to the nearest 50 MeV the energy shifts cannot be taken literally and although a few of the changes appear to be approaching 100 MeV it is fair to say that there is no major modification in the energies; other than the raising of some degeneracies and the reversal of a few levels. We shall now proceed to discuss some individual cases. The only ones for which there is corresponding P-matrix data (in table 2) are the $\frac{1}{2}^-$ primitives and we begin with those.

(*a*) $J^P = \frac{1}{2}^-$

Obviously, Jaffe's approximation does not induce mixing for one-dimensional matrices although there may be inaccuracies with the averaging procedure used, and thus the exotic (0, 1) $^2\mathscr{B}$ $o^4\bar{s}$ primitive at 1717 MeV (and $m_s R_0 = 1.615$) is a pure $\overline{\underline{18}}$ state. It is found to comprise 25.0 % NK, 8.3 % NK*, 25.0 % $\mathbf{N}_{\frac{1}{2}}\cdot\mathbf{K}$, 8.3 % $\mathbf{N}_{\frac{1}{2}}\cdot\mathbf{K}^*$ and 33.3 % $\mathbf{N}_{\frac{3}{2}}\cdot\mathbf{K}^*$ (i.e. 66.7 % colour octet components; where $\mathbf{N}_{\frac{1}{2}}$ and $\mathbf{N}_{\frac{3}{2}}$ are colour octet excitations of the nucleon with spin $\frac{1}{2}$ and $\frac{3}{2}$ respectively). (Roiesnel writes $\mathbf{N}_{\frac{3}{2}}$ as $\mathbf{N}^*$ but this style of notation for other baryon (flavour) octet members could lead to confusion with the colour octet excitations of the baryon decuplet, which can occur only as spin $\frac{1}{2}$ and not as spin $\frac{3}{2}$.) The open channel projection is $\xi_{NK} = 1$ and we expect a P-matrix pole in this channel at 1.72 GeV (above the compensation energy, signalling a negative phase shift) and with a predicted residue 0.14 GeV^3. We find good agreement with table 2; the discrepancy in the pole position being only 0.04. Although the residue is again too small, the correction to $\langle\Lambda\rangle$, given in (2.24), increases the residue prediction to 0.18 GeV^3 which is about the right amount (Roiesnel 1979). (Notice that the s quark mass correction is of less significance where high mass primitives are concerned.)

In the (1, 1) channel $^2\mathscr{A}$ matrix exhibits only small mixing and the eigenstates turn out to be 99.9 % pure $\underline{45}$ and $\underline{45}^*$ states. The lowest primitive is at 1905 MeV and $m_s R_0 = 1.688$. It is found to comprise 16.1 % NK, 10.1 % NK*, 7.1 % ΔK*, 0.8 % $\mathbf{N}_{\frac{1}{2}}\cdot\mathbf{K}$, 36.1 % $\mathbf{N}_{\frac{1}{2}}\cdot\mathbf{K}^*$, 3.6 % $\mathbf{N}_{\frac{3}{2}}\cdot\mathbf{K}^*$, 16.1 % $\mathbf{\Delta}_{\frac{1}{2}}\cdot\mathbf{K}$ and 10.1 % $\mathbf{\Delta}_{\frac{1}{2}}\cdot\mathbf{K}^*$ (i.e. 66.7 % colour octet components). Note that if this had been a pure $\underline{45}$ flavour state then its decomposition would have been 16.2 %, 9.5 %, 7.6 %, 1.1 %, 36.1 %, 3.8 %, 16.2 % and 9.5 % into each of the respective components. Thus some components are seen to be rather sensitive to mixing. Even so, Strottman's (1979) decomposition contradicts that found here (Bickerstaff 1982). The current calculation predicts the open channel projections $\xi_{NK} = 0.78$ and $\xi_{NK^*} = 0.62$. The other primitive is found at 2110 MeV and

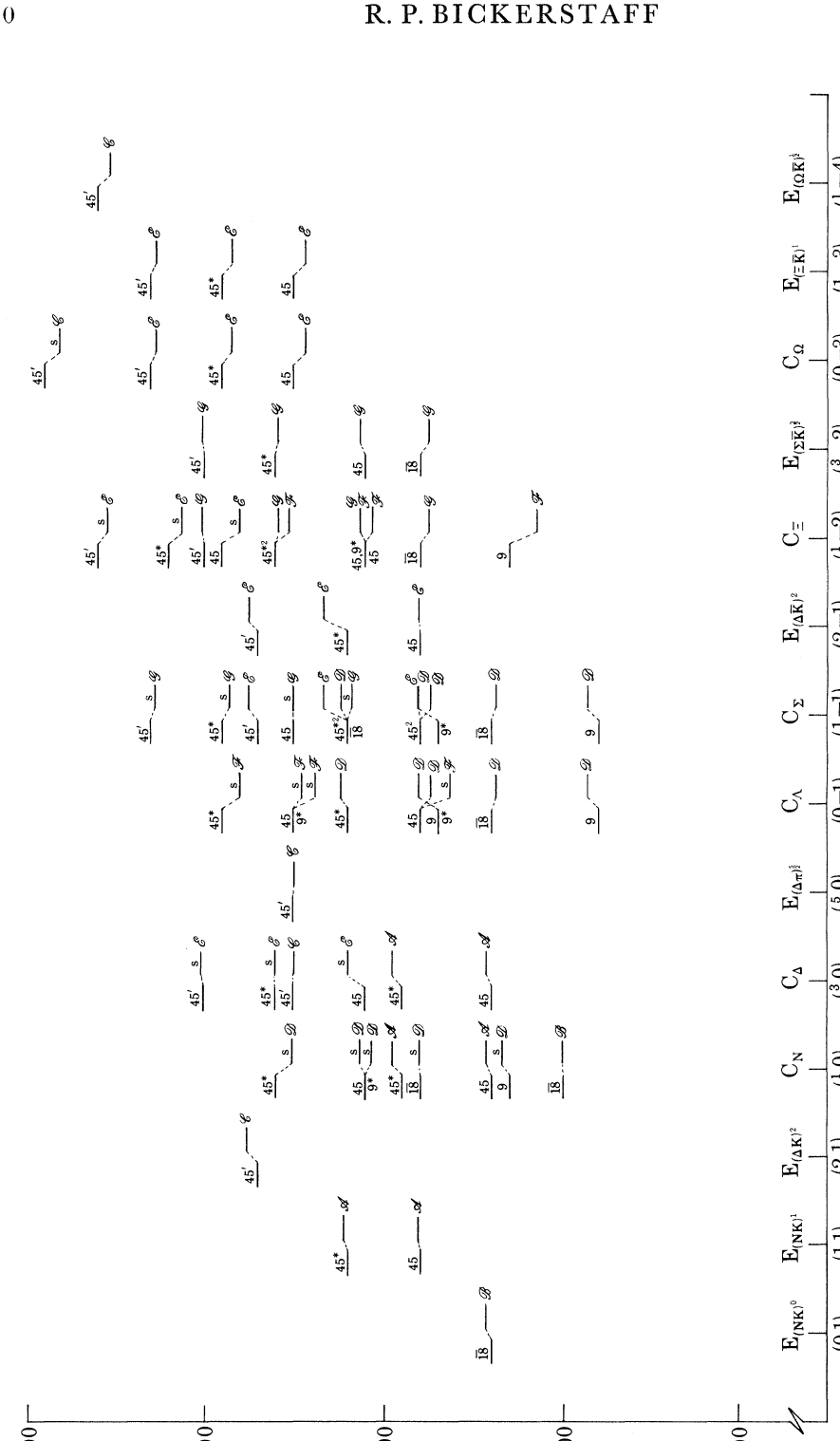

FIGURE 2. $q^4\bar{q}$ S-wave primitives. The channels are labelled both by their isospin and strangeness quantum numbers and a notation (Roiesnel 1979) which depicts whether it is exotic or cryptoexotic. For each channel the results on the left are those of Strottman (1979) rounded to the nearest 50 MeV) while those on the right are the current results, incorporating the effects of mixing induced by the flavour dependence of the colour-magnetic interaction strengths. Strottman's states are labelled with their SU_3^{fl} quantum numbers in dimensional notation while the improved states display the mixing matrix from which they derive. The presence of a hidden $s\bar{s}$ pair is indicated with the letter s, which enables the specific flavour content of the state to be determined.

(*a*) $J^P = \frac{1}{2}^-$. (*b*) $J^P = \frac{3}{2}^-$. (*c*) $J^P = \frac{5}{2}^-$.

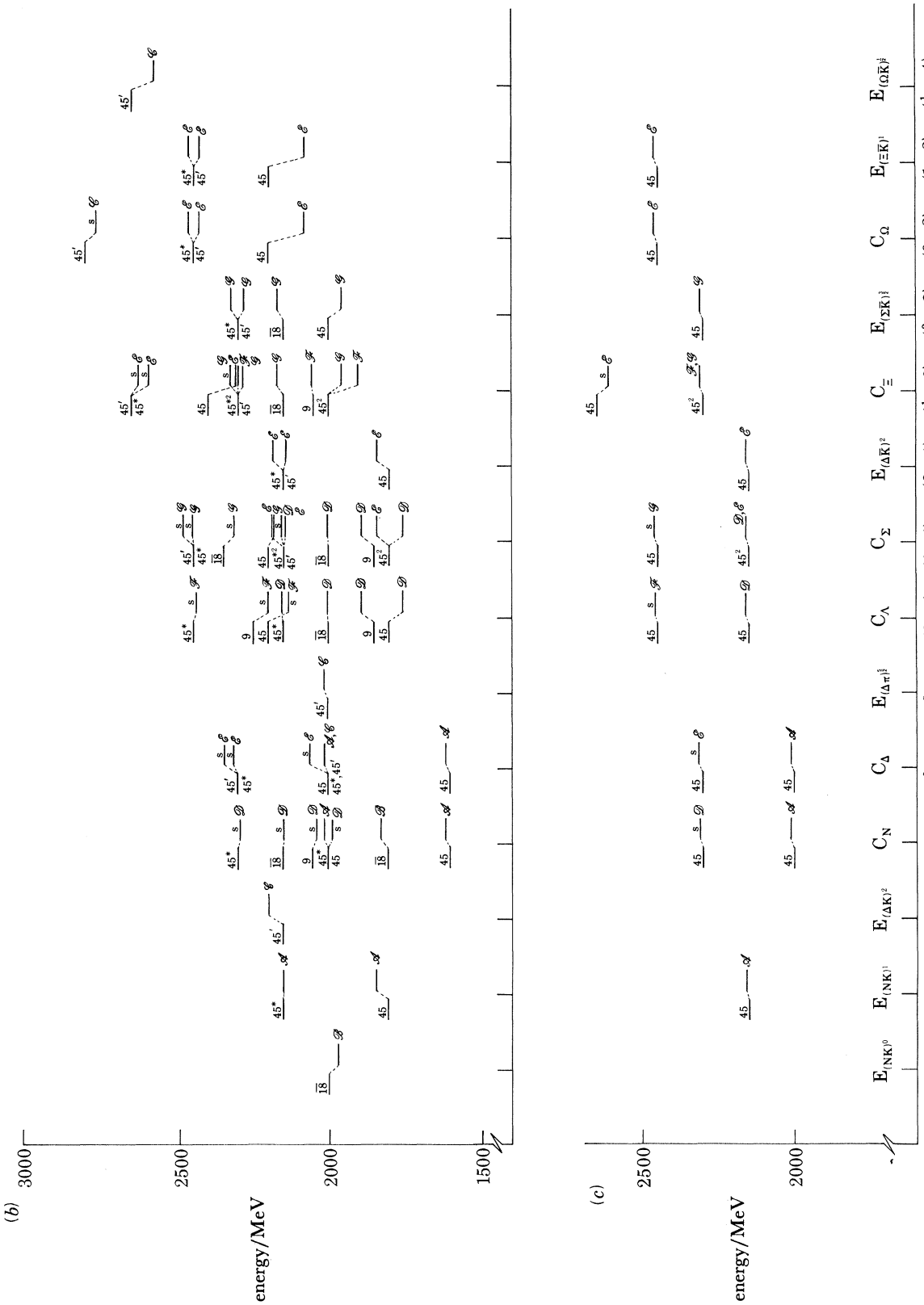

FIGURE 2 (*b*, *c*). For description see opposite.

$m_s R_0 = 1.770$. It comprises 0.6 % NK, 17.6 % NK*, 15.1 % ΔK*, 40.9 % $\mathbf{N}_{\frac{1}{2}} \cdot \mathbf{K}$, negligible $\mathbf{N}_{\frac{1}{2}} \cdot \mathbf{K}^*$, 7.6 % $\mathbf{N}_{\frac{3}{2}} \cdot \mathbf{K}^*$, 0.6 % $\mathbf{\Delta}_{\frac{1}{2}} \cdot \mathbf{K}$ and 17.6 % $\mathbf{\Delta}_{\frac{1}{2}} \cdot \mathbf{K}^*$ and thus has the open channel projections $\xi_{NK} = 0.18$ and $\xi_{NK^*} = 0.98$. One notes that these primitives are somewhat higher than the poles listed in table 2 but the data there only include the KN channel and therefore those poles are for an effective P matrix. Therefore (Jaffe & Low 1979) one does in fact expect the true poles to be higher than those in table 2 (cf. Corden *et al.* 1982).

In the cryptoexotic $(\frac{1}{2}, 0)$ channel there is moderate mixing within the $^2\mathscr{D}$ matrix. The close $\underline{9}^*$ and $\underline{45}$ are the most affected with the final states being about 92 % pure. The others are about 99 % pure with the $\underline{9}$ being the least affected. Of course the Jaffe approximation is exact for the $^2\mathscr{A}$ and $^2\mathscr{B}$ states. Consider the $^2\mathscr{B}$ state which is the lightest primitive in this channel, with mass 1502 MeV. It comprises 18.7 % Nπ, 6.3 % Nη$_0$, 2.1 % Nω$_0$, 6.3 % Nρ, 18.7 % $\mathbf{N}_{\frac{1}{2}} \cdot \pi$, 6.3 % $\mathbf{N}_{\frac{1}{2}} \cdot \boldsymbol{\eta}_0$, 2.1 % $\mathbf{N}_{\frac{1}{2}} \cdot \boldsymbol{\omega}_0$, 6.3 % $\mathbf{N}_{\frac{1}{2}} \cdot \boldsymbol{\rho}$, 8.3 % $\mathbf{N}_{\frac{3}{2}} \cdot \boldsymbol{\omega}_0$ and 25.0 % $\mathbf{N}_{\frac{3}{2}} \cdot \boldsymbol{\rho}$. (This decomposition should be compared with that found previously for an $\overline{18}$ state.) Both the Nπ and Nη channels are open and an effective pole in the Nπ channel can be expected below the Nη threshold; as one indeed finds.

In the $(\frac{3}{2}, 0)$ channel, small mixing occurs within the $^2\mathscr{E}$ matrix with its eigenstates being about 99.8 % pure. The other states are exactly described by Jaffe's approximation. The lightest primitive is the $^2\mathscr{A}$ state at 1713 MeV. It comprises 16.2 % Nπ, 9.5 % Nρ, 2.9 % Δω$_0$, 4.8 % Δρ, 1.1 % $\mathbf{N}_{\frac{1}{2}} \cdot \pi$, 36.1 % $\mathbf{N}_{\frac{1}{2}} \cdot \boldsymbol{\rho}$, 3.8 % $\mathbf{N}_{\frac{3}{2}} \cdot \boldsymbol{\rho}$, 6.1 % $\mathbf{\Delta}_{\frac{1}{2}} \cdot \boldsymbol{\eta}_0$, 10.2 % $\mathbf{\Delta}_{\frac{1}{2}} \cdot \pi$, 3.6 % $\mathbf{\Delta}_{\frac{1}{2}} \cdot \boldsymbol{\omega}_0$ and 5.9 % $\mathbf{\Delta}_{\frac{1}{2}} \cdot \boldsymbol{\rho}$. This should be compared with the decomposition quoted earlier for a pure $\underline{45}$ flavour state. Both the Nπ and Nρ channels are open and we can expect an effective pole in the Nπ channel, below the Nρ threshold. This agrees qualitatively with table 2 although the pole position quoted there is very uncertain. According to (2.24) the Δπ channel is also open and this seems to correspond with the data (Roiesnel 1979).

The $(0, -1)$ channel contains some interesting examples of mixing. In the $^2\mathscr{D}$ matrix the mixing between the $\underline{9}^*$ and $\underline{45}$ states is so strong that their order becomes reversed. In fact the lower of these levels is found to contain only about 15 % $\underline{9}^*$ and 85 % $\underline{45}$ with the next level the reverse of this. The other three eigenstates are not unduly affected by mixing and in order of increasing energy are about 99.9 % $\underline{9}$, 98.7 % $\overline{18}$ and 99.5 % $\underline{45}^*$. The lowest $^2\mathscr{F}$ state is almost pure $\underline{9}$ but the $\underline{9}^*$ and $\underline{45}$ mix with each other to a moderate degree and the eigenstates are only about 92 % pure. The highest state only mixes a little and is 99 % $\underline{45}^*$. Consider the lowest lying primitive in this channel which is the $^2\mathscr{D}$ $o^3s\bar{o}$ flavour $\underline{9}$ state at 1432 MeV and $m_s R_0 = 1.497$. It comprises 19.6 % Σπ, 11.5 % $N\bar{K}$, 1.9 % Λη$_0$, 0.1 % $N\bar{K}^*$, 0.1 % Λω$_0$, 0.1 % Σρ, a negligible amount of Σ*ρ and 66.7 % colour octet components. (Those colour octet components that are present may be inferred from the colour neutral components. Anybody requiring the explicit decomposition is invited to contact the author.) The presence of some Σ* indicates contamination from the $\underline{45}$ (since in the Jaffe approximation only $\underline{45}$ and $\underline{45}'$ can couple to the baryon decuplet (Roiesnel 1979) but the strongest mixing is with the $\overline{18}$. In the Jaffe limit the $N\bar{K}$ component grows by about 1 % at the expense of the Σπ and similarly the Λη$_0$ and Λω$_0$ components alter a little. However, as claimed previously (Bickerstaff 1982), there is still major variance with Strottman's (1979) work. This primitive lies almost on top of the $N\bar{K}$ threshold and we shall take this channel to be open. Thus we predict a pole at 1.43 GeV with parameters

$$\lambda = 0.12\,\mathrm{GeV}^3, \quad \xi_{N\bar{K}} = 0.61, \quad \xi_{\Sigma\pi} = 0.79;$$

cf. Roiesnel (1979) who, unfortunately, used Strottman's (1979) results. The agreement with table 2 is reasonable although this time λ is too large and the channel couplings are opposite in

strength to those found by Roiesnel (1979). (Without the s quark mass correction the residue would be an even larger 0.14 GeV³.) As discussed by Roiesnel this primitive can also be seen as a pole in the reduced single-channel $\pi\Sigma$ P matrix at about 1.41 GeV with a very small residue and hence it may be associated with the $\Lambda(1405)$ effect (cf. Dalitz & McGinley 1981; Oades & Rasche 1982; Dalitz *et al.* 1982.) This is similar to the suggestion that some $q^2\bar{q}^2$ primitives may be associated with the S*(980) and δ(980).

In the $(1, -1)$ channel the mixing within the $^2\mathscr{E}$ and $^2\mathscr{G}$ matrices is fairly small with the eigenstates mostly being more than 99 % pure. The only exceptions are the two lowest $^2\mathscr{G}$ states which are a little under 99 % pure. Of course the $^2\mathscr{D}$ matrix is degenerate with that in the $(0, -1)$ channel. (It should be obvious that this degeneracy has nothing to do with the Jaffe approximation.) The lowest primitive comprises 1.9 % $\Lambda\pi$, 13.1 % $\Sigma\pi$, 11.5 % $N\bar{K}$, 6.5 % $\Sigma\eta_0$, 0.1 % $N\bar{K}$*, 0.1 % $\Lambda\rho$, 0.1 % $\Sigma\rho$, a negligible amount of $\Sigma\omega_0$, $\Sigma^*\rho$ and $\Sigma^*\omega_0$ and 66.7 % colour octet components. Both the $\Lambda\pi$ and $\Sigma\pi$ channels are open and again this primitive is located on the $N\bar{K}$ threshold. Roiesnel (1979) attempted a P-matrix analysis for this primitive but had to extrapolate the amplitudes till 1.54 GeV to find a pole and no reliable analysis of its parameters was possible. Evidently the degeneracy with the $I = 0$ primitive must be broken but although annihilation of the ω_0 subsystem component into a transverse magnetic gluon (Jaffe 1977*a*; Bickerstaff & McKellar 1982; Barnes *et al.* 1982) is expected to give a shift of the correct sign, it is not clear that that process alone could account for a 90 MeV splitting, cf. Högaasen & Wroldsen (1982).

In the $(\frac{1}{2}, -2)$ channel the $^2\mathscr{F}$ matrix displays strong mixing. Although the lowest and highest eigenstates are relatively pure $\underline{9}$ and $\underline{45}$* respectively, the $\underline{9}$* and $\underline{45}$ states mix so strongly that once more we find an example where the resulting levels are the reverse of what is the case in the Jaffe approximation. The lower of these turns out to be only about 25 % $\underline{9}$* compared to 75 % $\underline{45}$ with the other being the (approximately) orthogonal combination. In contrast the mixing in the $^2\mathscr{E}$ matrix is fairly small and mixing in the $^2\mathscr{G}$ matrix is not much larger, at about the 1 to 2 % level.

In the $(0, -3)$ channel, mixing is again small in the $^2\mathscr{E}$ matrix with the lowest eigenstate being almost pure $\underline{45}$. The $\underline{45}$* and $\underline{45}'$ levels only mix a little with each other and the final eigenstates are about 99.6 % pure.

(*b*) $J^P = \frac{3}{2}^-$

This brings us to mixing in the $\frac{3}{2}^-$ primitives where the most notable features revolve around the degeneracy of the $\underline{45}$* and $\underline{45}'$ states in the Jaffe limit. Let us proceed with a channel-by-channel discussion.

In the $(1, 1)$ channel, mixing within the $^4\mathscr{A}$ matrix is fairly small and the eigenstates are about 99.9 % pure. Of course in the $(\frac{1}{2}, 0)$ channel the Jaffe approximation for this matrix is exact, the specific configuration only containing o quarks. However the $^4\mathscr{D}$ matrix in this channel does exhibit moderate mixing. Its lowest eigenstate is about 94 % $\underline{45}$ and 6 % $\underline{9}$, the next is about 91 % $\underline{9}$, 6 % $\underline{45}$ and 3 % $\underline{\overline{18}}$, the next is about 96 % $\underline{\overline{18}}$ and 3 % $\underline{9}$ while the highest eigenstate is a relatively unscathed $\underline{45}$* state of about 99.7 % purity.

In the $(\frac{3}{2}, 0)$ channel the degeneracy of the $^4\mathscr{A}$ $\underline{45}$* and the $^4\mathscr{C}$ $\underline{45}'$ states is clearly exact. There is though some mixing within the $^4\mathscr{E}$ matrix. The lowest eigenstate is 99.8 % $\underline{45}$ but the degeneracy of the $\underline{45}$* and the $\underline{45}'$ is raised by 30 MeV and the lower of the two eigenstates turns out to be only 79 % $\underline{45}$*, and 21 % $\underline{45}'$, with no apparent radial variation. The other eigenstate is the reverse of this and does display some radial variation, but only very small. We therefore conclude that even in the Jaffe limit the $\underline{45}$* and $\underline{45}'$ must be regarded as strongly mixed.

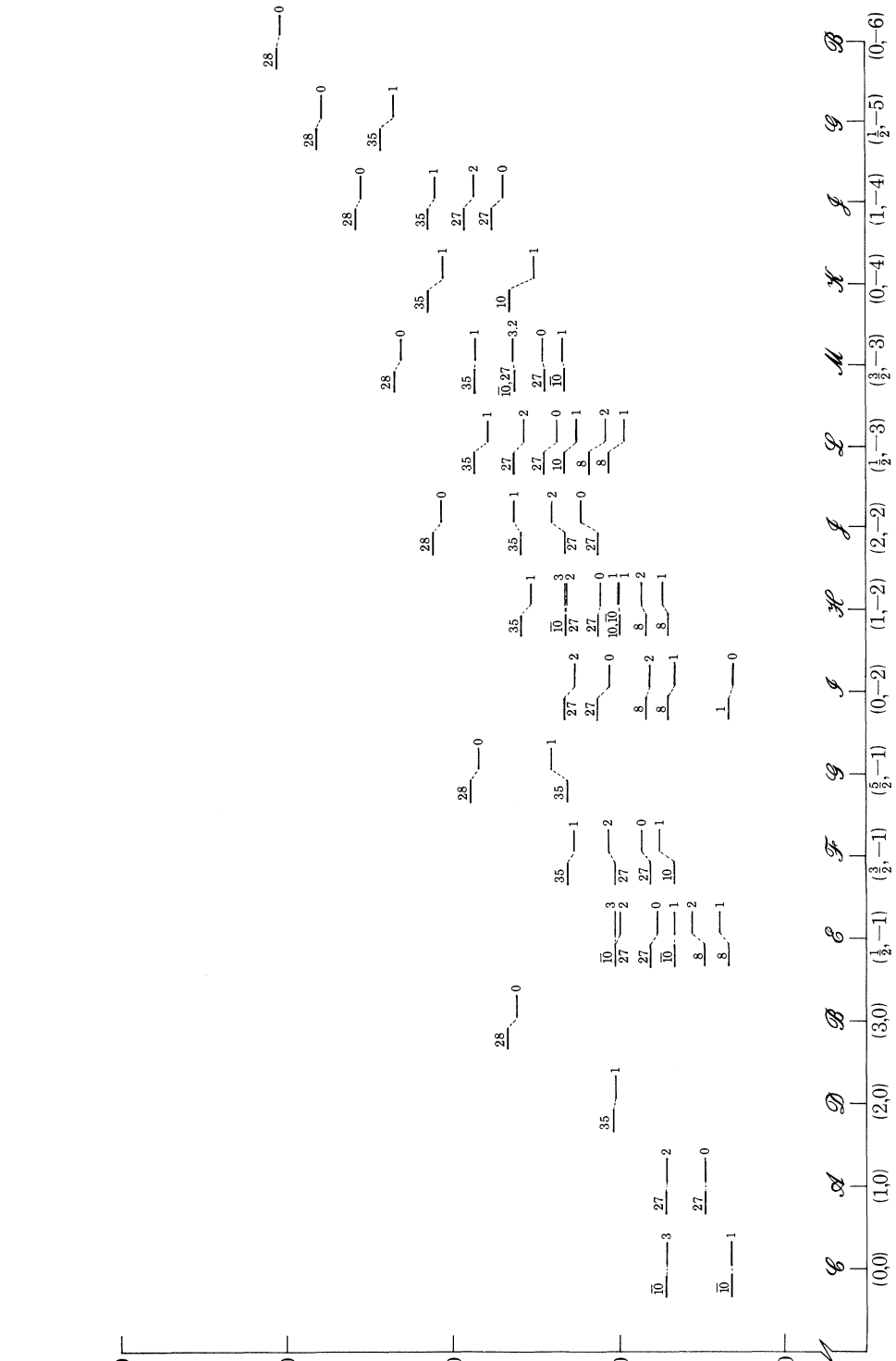

FIGURE 3. q^6 S-wave primitives. The channels are labelled by their isospin and strangeness quantum numbers. For each channel the results on the left are from Mulders *et al.* (1980) while those on the right are the current results, which incorporate the effects of mixing induced by the flavour dependence of the colour-magnetic interaction strengths. The results from Mulders *et al.* are labelled with their SU_3^f quantum numbers in dimensional notation while the improved states display the spin quantum number. All states in the same channel derive from colour-spin matrices with a common name and this is shown with the channel labels.

In the $(0, -1)$ channel both the $^4\mathscr{D}$ and $^4\mathscr{F}$ matrices exhibit moderate mixing. The lowest eigenstate is about 99 % $\underline{45}$. (Because Strottman's (1979) techniques for dissociating this state unfortunately cannot be trusted (Bickerstaff 1982) it is perhaps worth stating that this comprises 27.8 % $\Sigma^*\pi$, 3.3 % $N\bar{K}^*$, 2.2 % $\Lambda\omega_0$, a negligible amount of $\Sigma\rho$ and $\Sigma^*\rho$, and 66.7 % colour octet components.) The next state is about 95 % $\underline{9}$ with the strongest admixture apparently being $\underline{\overline{18}}$, of which there is about 4 %. Next we have an $\underline{\overline{18}}$ state of about 96 % purity with 4 % $\underline{9}$ while the highest $^4\mathscr{D}$ state is fairly pure $\underline{45}^*$ with only about 0.2 % admixtures. The lowest $^4\mathscr{F}$ state is about 95 % $\underline{45}$ and 4 % $\underline{9}$ with the next state being roughly opposite to this and the highest being about 99.8 % $\underline{45}^*$.

In the $(1, -1)$ channel the $^4\mathscr{E}$ matrix displays a little mixing. The lowest eigenstate is 99.6 % $\underline{45}$ but the degeneracy of the $\underline{45}'$ and $\underline{45}^*$ is split by about 40 MeV and the lower eigenstate is 83 % $\underline{45}'$ and 16 % $\underline{45}^*$. The other is the reverse of this. Because the radial variation in these admixtures is small a very similar mixture (in fact about 84 % and 16 %) must be assumed in the Jaffe limit. There is similar mixing in the $^4\mathscr{G}$ matrix. Its lowest two eigenstates are more than 99 % pure but the highest two are strong admixtures of $\underline{45}^*$ and $\underline{45}'$ with the degeneracy of these states being raised. Again there is no strong radial variation in the admixtures.

In the $(\frac{1}{2}, -2)$ channel the $^4\mathscr{F}$ matrix exhibits only small mixing with the lowest two states being the most affected but nevertheless still about 99 % pure. The description of mixing in the $^4\mathscr{G}$ matrix is similar to that given for the $(1, -1)$ channel case; although the specific configuration and the precise details here are of course different. Mixing within $^4\mathscr{E}$ is also similar to the $(1, -1)$ channel case. The lowest eigenstate is 99.7 % $\underline{45}$, the next is 84 % $\underline{45}^*$ and 16 % $\underline{45}'$, with the remaining one the reverse of this. The same remarks concerning the Jaffe limit apply.

In the $(0, -3)$ channel the $^4\mathscr{E}$ matrix displays mixing similar to that in the $(\frac{3}{2}, 0)$ channel. The lowest eigenstate is 99.7 % $\underline{45}$, the next is 79 % $\underline{45}'$ and 21 % $\underline{45}^*$ and the last is 79 % $\underline{45}^*$ and 21 % $\underline{45}'$. As in the $(\frac{3}{2}, 0)$ case this last eigenstate has no radial variation but the reason is not obvious. (Note that the order of the highest two eigenstates is reversed in the present case.)

(c) $J^P = \frac{5}{2}^-$

All of the $\frac{5}{2}^-$ matrices are one-dimensional. We note only that the degeneracy of the $^6\mathscr{D}$ and $^6\mathscr{E}$ $(1, -1)$ $\underline{45}$ states is exact as also is the degeneracy of the $^6\mathscr{F}$ and $^6\mathscr{G}$ $(\frac{1}{2}, -2)$ states; but the isospin degeneracy between the $(\frac{1}{2}, 0)$ $^6\mathscr{D}$ and $(\frac{3}{2}, 0)$ $^6\mathscr{E}$ states is raised as also is the degeneracy between the $(0, -1)$ $^6\mathscr{F}$ and $(1, -1)$ $^6\mathscr{G}$ states.

6. q^6 PRIMITIVES

The matrix elements of the colour-spin operator for this configuration have been tabulated in (Bickerstaff 1980). A few were also given in (Bickerstaff & Wybourne 1980). A summary of the matrices arising for each generic configuration is given in table 5. The largest is the 4×4 $^3\mathscr{H}$. Notice that for the $^1\mathscr{J}$ and $^3\mathscr{K}$ matrices in the o^2s^4 configuration one again needs to check for permutation phase changes when using matrices derived for the generic a^4b^2 configuration. (Of course there is no necessity for this check in one-dimensional matrices.) In Jaffe's approximation the colour-spin operator would be diagonal if the basis vectors had good SU_6^{CJ} symmetry for the combined six-quark system. However, as with the $q^4\bar{q}$ configuration, this block-diagonalization does not take place for the matrices given in (Bickerstaff 1980) because, to aid the evaluation of the matrix elements (Bickerstaff & Wybourne 1980) they are given in a basis where the six quarks

only have good $SU_2^J \times SU_3^C$ symmetry. This notwithstanding, the known (Jaffe 1977*d*; Mulders *et al.* 1980; Wybourne 1978) eigenvalues in Jaffe's approximation provide a useful check on the matrices. (Beware that the eigenvalues given by Mulders *et al.* (1980) have a different normalization.) The eigenvectors in Jaffe's approximation may once more be labelled by their associated flavour multiplets. This time though the SU_3^{fl} representations are irreducible. We can use the following (standard) dimensional notation: $\underline{1} \equiv \{0\}$, $\underline{8} \equiv \{21\}$, $\underline{10} \equiv \{3\}$, $\overline{\underline{10}} \equiv \{3^2\}$, $\underline{27} \equiv \{42\}$, $\underline{35} \equiv \{51\}$ and $\underline{28} \equiv \{6\}$. Note that in Jaffe's approximation the $J^P = 1^+$ $\underline{10}$ and $\overline{\underline{10}}$ states are degenerate, as are the 2^+ $\underline{27}$ and 3^+ $\overline{\underline{10}}$.

TABLE 5. q^6 COLOUR–SPIN MATRICES

(The SU_6^{CJ} quantum numbers of the quark subsystems in the generic configuration are denoted by Schur functions. Where applicable, the arbitrary labels used in (Bickerstaff & Wybourne 1980) are shown in parentheses beside the dimensions.)

matrix name	generic configuration	internal SU_6^{CJ} quantum numbers	dimension in $J = 0$	dimension in $J = 1$	dimension in $J = 2$	dimension in $J = 3$
$\mathscr{A}$	q_a^6	$\{2^2 1^2\}$	1	—	1	—
$\mathscr{B}$	—	$\{0\}$	1	—	—	—
$\mathscr{C}$	—	$\{2^3\}$	—	1	—	1
$\mathscr{D}$	—	$\{21^4\}$	—	1	—	—
$\mathscr{E}$	$q_a^5 q_b$	$\{2^2 1\}\{1\}$	1	2	2	1
$\mathscr{F}$	—	$\{21^3\}\{1\}$	1	2	1	—
$\mathscr{G}$	—	$\{1^5\}\{1\}$	1	1	—	—
$\mathscr{H}$	$q_a^4 q_b^2$	$\{21^3\}\{1^2\}$	1 (b)	4 (a)	2 (c)	1 (d)
$\mathscr{I}$	—	$\{2^2\}\{1^2\}$	2 (e)	1 (f)	2 (g)	—
$\mathscr{J}$	—	$\{1^4\}\{1^2\}$	2 (h)	1 (i)	1 (j)	—
$\mathscr{K}$	—	$\{1^4\}\{2\}$	—	2 (k)	—	—
$\mathscr{L}$	$q_a^3 q_b^3$	$\{21\}\{1^3\}$	1	3	2	—
$\mathscr{M}$	—	$\{1^3\}\{1^3\}$	2	2	1	1

A comparison of the q^6 primitives in Jaffe's approximation with the current calculation is presented in figure 3. Unfortunately Wybourne (1978) only gives the states associated with the lowest spin for each given SU_3^{fl} multiplet and Mulders *et al.* (1980, see also Mulders 1980) only tabulate the states with strangeness, $S = 0, -1$ and -2. However, it is easy to use the methods of Mulders *et al.* (1980) to obtain all states and it is the results obtained by doing this which are used in figure 3. Although these results have not been rounded, they come from a simplified model and the quantitative energy shifts must be regarded cautiously. Notice that all spin states are included in the same figure. The most notable feature of removing Jaffe's approximation is the raising of degeneracies between channels with different isospin but the same strangeness. For example the ${}^1\mathscr{I}$, ${}^1\mathscr{H}$ and ${}^1\mathscr{J}$ $\underline{27}$ states with $S = -2$ are all degenerate in the Jaffe limit. The raising of this degeneracy is of the same nature as the Λ–Σ splitting in the ordinary baryons (DeGrand *et al.* 1975) and is due to all universal averaging procedures for the interaction strengths being inaccurate. Similar cases did arise in the $q^4\bar{q}$ configuration, but not to the extent seen here. (Recall that most of the degeneracies in the $q^2\bar{q}^2$ and $q^4\bar{q}$ sectors, possessing the above features, had a different origin and were not raised by removing the Jaffe approximation.) Mixing between different flavour multiplets is very small in the q^6 sector and all states have less than 1% impurity admixtures. The only possible exception to this is the $J^P = 1^+$ degenerate $\underline{10}$ and $\overline{\underline{10}}$ pair in the $(1, -2)$ channel. This degeneracy is raised by a meagre 1 MeV but it is difficult to ascertain just what the pure $\underline{10}$ and $\overline{\underline{10}}$ wave functions are and therefore the mixing remains undetermined. However, the $\underline{8}$ and $\underline{35}$ states in the same ${}^3\mathscr{H}$ matrix are not much affected.

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The only primitives for which there is available P-matrix data are the $^3\mathscr{C}$ and $^1\mathscr{A}$ states in the (0, 0) and (1, 0) channels at 2165 and 2243 MeV respectively. Both states are properly described in Jaffe's approximation and this work has little to add; other than to briefly review the current situation. The first of these states has the same quantum numbers as the deuteron. However it has nothing to do with that bound state. Indeed it does not really resemble it; it being comprised of 11 % NN, 9 % $\Delta\Delta$ and 80 % colour octet components (Matveev & Sorba 1978; Harvey 1981, Mulders 1982*a*). (The $^1\mathscr{A}$ state has the same content.) Further, it lies above the NN threshold and even above the NN compensation energy. Actually this is quite encouraging. It means that at this energy the NN phase shift is predicted to be negative and this corresponds (Simonov 1981; Mulders 1982*a*, *b*) with the well known repulsive core in the NN interaction. The deuteron in fact corresponds to a zero in the P matrix, lying just below threshold (Jaffe & Shatz 1980; Simonov 1981). Nevertheless, P-matrix poles are found in reasonable agreement with the predictions; the discrepancies in the pole positions being $\Delta = 0.15$ and 0.27 for the $^3S_1$ $I = 0$ and $^1S_0$ $I = 1$ channels respectively. However, the predicted residues, which are 0.11 GeV^3 in both cases, are much too small and the splitting between the $^3S_1$ and $^1S_0$ poles is smaller than predicted (Jaffe & Shatz 1980).

The lowest primitive in the $(\frac{1}{2}, -1)$ channel is a 1^+ flavour $\underline{8}$ state at 2196 MeV and $m_s R_0 = 1.774$. It mixes only to a small degree with the 1^+ flavour $\underline{\overline{10}}$ primitive, at 2331 MeV and $m_s R_0 = 1.826$; of which there is about a 0.2 % admixture. Unfortunately the isoscalar factors necessary to calculate the baryon–baryon content of these primitives are not yet avilable. (We can note though that all q^6 primitives contain 80 % colour octet components and 20 % ordinary baryons (Bickerstaff *et al.* 1982*b*; Mulders 1982*a*). Incidentally, this fact demonstrates that the deuteron is not totally antisymmetric under exchange of quarks between nucleons.) It is interesting to ponder whether there may be some connection with the well established resonance at 2128 MeV in this channel (Jaffe 1982*a*).

The (0, −2) channel is of some interest. The lowest primitive is a 0^+ flavour $\underline{1}$ at 2154 MeV and $m_s R_0 = 1.693$. Because the 0^+ $\underline{27}$ state at 2527 MeV and $m_s R_0 = 1.837$ is so far removed, mixing with it is so small as to be barely detectable. One finds that the composition of this lowest primitive is 2.5 % $\Lambda\Lambda$, 10.0 % NΞ, 7.5 % $\Sigma\Sigma$, negligible $\Sigma^*\Sigma^*$ and 80 % colour octet components (Bickerstaff & Wybourne 1981). As Jaffe (1977*d*) has pointed out, this state is below the $\Lambda\Lambda$ threshold and ought even to be stable against strong decay. The accuracy of this prediction of course depends on the validity of the bag model description of such a state. Since the primitive is below all thresholds the relevant quantity to consider is the actual mass of the supposed bound state, i.e. the P-matrix pole position when the artificial confinement (in unconfined channels) is removed to infinity. While doing this can only lower the pole position – perhaps by as much as 100–200 MeV (Soldate 1981) – adding further terms to the bag Hamiltonian (with possible accompanying changes in α_c) could raise the final value. In particular it is likely that pionic corrections increase the mass (Mulders & Thomas 1982). Also, altering the shape of the potential inside the artificial confining boundary could raise the mass; although this would perhaps run contrary to the spirit of the bag model whose square-well potential is inspired by a supposed abrupt transition from a perturbative to a confining phase of the QCD vacuum. Furthermore, note that since it is the actual mass which is important a centre-of-mass correction to this should be applied. It appears that this also will increase the mass (Liu & Wong 1982). The question of the existence of this stable dihyperon in the bag model obviously deserves further theoretical investigation. It is not experimentally ruled out (Jaffe 1981, 1982*a*; Aerts & Dover 1982). Whether or not it

exists though, it is perhaps worth pointing out that because of its apparently small coupling to the $\Lambda\Lambda$ channel it is probably quite misleading to regard it as a $\Lambda\Lambda$ bound state.

The next lowest state in the $(0, -2)$ channel is a flavour $\underline{8}$ state at 2329 MeV and $m_s R_0 = 1.764$. It comprises (Bickerstaff & Wybourne 1981) 8.9% NΞ, 4.4% NΞ^* and 6.7% $\Sigma\Sigma^*$ with the remaining components being colour octet combinations. Thus it should appear as a P-matrix pole in the NΞ channel, below the compensation energy indicating an attractive potential. The predicted residue is 0.09 GeV^3.

7. Conclusions

The consequences of removing Jaffe's approximation for the colour hyperfine term are two-fold. Firstly, mixing may be induced among the approximate eigenstates and secondly, even when there is no mixing, the eigenenergies may be altered because there is no averaging procedure for the interaction strengths which can be universally valid. The only time that Jaffe's approximation is exact is when all quarks are of the same flavour. Nevertheless we have found that the approximation still provides a fair description of the energy spectrum. The most notable changes are the raising of some degeneracies and a switch in the order of a few levels. However, nowhere in the $q^2\bar{q}^2$, $q^4\bar{q}$ and q^6 configurations did we find any case where the actual change in energy was more than 100 MeV. Considering the reliability of the current bag model (DeGrand *et al.* 1975) this is quite acceptable. On the other hand mixing of the eigenstates can result in large changes in their content. As is usual in any configuration mixing, the greatest effects are found where the unmixed states are close in energy. We have been able to identify all those cases where mixing is strong. Of particular importance are those cases where degeneracy exists. Unless mixing raises this degeneracy the eigenstates cannot be predicted. For example we found some 2^+ $q^2\bar{q}^2$ primitives which are radically different from the flavour symmetric states assumed by Jaffe (1977*a*) and which have been suggested (Li & Liu 1982; Achasov *et al.* 1982) as candidates for structure seen in the two photon production of hadrons. Even where mixing is small though we have found that the projections of the multiquark eigenstates onto hadronic channels can be rather sensitive to impurity admixtures.

Because the source of mixing treated here is rather weak one must conclude, in view of the fairly high density of primitives in some channels, that additional mixing terms, which have been neglected in this treatment, can further appreciably modify many of the eigenstates. Thus, the hadronic content of multiquark primitives cannot be regarded as a very sound prediction of the present model.

Fortunately, those few primitives for which there exist P-matrix data are low-lying (and consequently distant from most other states) and are not greatly influenced by the mixing discussed here. Further, we have not found any appreciable shifts in their energies and apart from some substantial corrections for the s quark mass our predictions are little different from earlier approximate treatments (Jaffe & Low 1979; Roiesnel 1979). (There are however changes in some residue predictions due to the lack of phase consistency in those works.) An attempt has been made to quantify the discrepancy between the predicted and experimental pole positions. Although the predictions seem reasonable, and are certainly of the right order of magnitude, the detailed agreement is by no means excellent. In particular it appears, on the basis of the seven cases where detailed comparison is possible, that the quantitative spread of the predictions about E_c is being consistently overestimated. One must hope that future refinements in both the bag model and the P-matrix formalism will improve the situation. At least the qualitative relation

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between primitive poles and the compensation energy is in agreement and this can be indirectly attributed in part to the colour-hyperfine interaction. On the whole, one does find P-matrix poles when expected but there may be a discrepancy in the $I = 0$ $\pi\pi$ and $K\bar{K}$ channels where there do not appear to be sufficient poles to accommodate both the $q^2\bar{q}^2$ primitives and the $q\bar{q}$ orbital excitations. However, better and more extensive data is required before this can be claimed to be a real problem.

On the subject of residues, the bag model predictions of Jaffe & Low (1979) are not very successful, although in fairness it must be recognized that their 'predictions' are little more than crude estimates (Jaffe & Low 1979). Of the nine residue predictions which can be compared directly with the data, eight seem too small and one is too large. Roiesnel's (1979) correction to the residues makes little difference for those $q^2\bar{q}^2$ primitives considered but does give substantial increases for the $q^4\bar{q}$ primitives. However, of the two $q^4\bar{q}$ residues which can be compared with the data one is already too large. (In Roiesnel's work it was too small because of his use of incorrect eigenfunctions.) On these grounds one could doubt the efficacy of his correction. Recent work (Bickerstaff 1983) has though gone some way towards rectifying residue predictions and the reader is referred there for possible cures. Concerning channel couplings, it was only possible in two cases – associated with the $S^*(980)$ and $\Lambda(1405)$ – to compare predicted channel couplings with those obtained in a two-channel P-matrix analysis of scattering data. Both predictions are in qualitative error concerning which channel has the larger coupling. This may be an indication that further mixing processes, for example pair anihilations and pionic corrections, are important.

Although Jaffe & Low (1979) have claimed tentative evidence for colour-hyperfine structure in the spectrum of primitives it is too early to judge from these observations the usefulness of bag model predictions for multiquark primitives. Both the bag model multiquark calculations and the P-matrix formalism (as applied to bag model multiquark states) are still in their infancy. At least there is not violent disagreement between the bag primitives and the P-matrix data. The P-matrix formalism has been criticized, for example, by Irving *et al.* (1981) for being an inefficient means of parametrizing the data. This criticism stems mainly from attempts to fit a P matrix to incomplete phase-shift data. Fortunately, Jaffe (1982*b*) has shown how to overcome that problem. In any case, one must not lose sight of the fact that it is quite remarkable that one can extract anything at all from a calculation which uses such an artificial (and most unphysical) boundary condition. At present the P matrix provides the only viable interpretation of the bag model multiquark eigenstates. There are though possibilities (W. R. Moreau, private communication; Simonov 1981; Aaron & Friedman 1982) for developing an approach more closely allied to the standard technique of Feshbach (1958, 1962) projection operators.

We conclude by commenting that whatever the accuracy of the current bag model calculations may be, the P-matrix analyses of scattering data do show good evidence for multiquark exotics. The implication though is that a proper theoretical treatment of multiquark systems must find that many, if not most, of the states are unbound and non-resonant. This in fact is the result of a recent calculation using a non-relativistic potential model (Weinstein & Isgur 1982).

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